External non-Abelian gauge fields for cold atoms

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Outline

Why effective magnetic field for atoms?

Atomic physics $\Longleftrightarrow$ Solid state physics:

- Degenerate Fermi gas $\Longleftrightarrow$ Electrons in solids
- Atoms in optical lattices

Advantages and disadvantages of cold atoms

- **Advantage:** Freedom in changing experimental parameters that are often inaccessible in standard solid state physics
- **Disadvantage:** Trapped atoms are electrically neutral particles. Direct analogy with magnetic properties of solids is not necessarily straightforward

Other uses of the effective magnetic field

Analogies with the elementary particle physics

Cold atomic gasses are an analog not only to the solid state physics. Creation of the effective gauge potentials allows for the motion of cold atoms to be described by equations that usually appear in the elementary particle physics.

- Non-Abelian gauge potentials
- Magnetic monopole
- Ultrarelativistic Dirac fermions
- Zitterbewegung
- Negative reflection

Magnetic field and rotation

- Coriolis force:

$$\mathbf{F}_C = 2m\mathbf{v} \times \Omega$$

- Lorenz force:

$$\mathbf{F}_L = q\mathbf{v} \times \mathbf{B}$$

Rotation is similar to the magnetic field.

Ways to create effective magnetic field for cold atoms

- Rotation — usual method to create effective magnetic field
  - Constant effective magnetic field $B_{\text{eff}} \sim \Omega$
  - Trapping frequency $\omega_{\text{eff}} = \omega - \Omega$
  - Effective magnetic field acts on atoms in the same way
- Optical lattices having assimetry in the atomic transitions between the lattice sites.
  - Abelian effective gauge potentials

  - D. Jaksch and P. Zoller, New J. Phys. **5**, 56 (2003)
  - E. Mueller, Phys. Rev. A **70**, 04163(R) (2004)
  - A. S. Sørenson, E. Demler, and M. D. Lukin, Phys. Rev. Lett. **94**, 086803 (2005)

  - Non-Abelian effective gauge potentials in optical lattices

  - K. Osterloh, M. Baig, L. Santos, P. Zoller, and M. Lewenstein, Phys. Rev. Lett. **95**, 010403 (2005)

Effective magnetic field induced by EIT

Effective gauge potentials can be created using light beams with non-zero relative orbital angular momentum (OAM) in the EIT configuration.

Advantages

- No rotation
- No need for optical lattice

Abelian gauge fields:

- G. Juzeliūnas and P. Öhberg, Phys. Rev. Lett. **93**, 033602 (2004).

Non-Abelian gauge fields:

- J. Ruseckas, G. Juzeliūnas, P. Öhberg, and M. Fleischhauer Phys. Rev. Lett. **95**, 010404 (2005)

Adiabatic Approximation

- The full atomic Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2M} + \hat{V}(\mathbf{r}) + \hat{H}_0(\mathbf{r}, t).$$

  - $\hat{H}_0(\mathbf{r}, t)$ — the Hamiltonian for the electronic (fast) degrees of freedom,
  - $\hat{p}^2/2M + \hat{V}(\mathbf{r})$ — the Hamiltonian for center of mass (slow) degrees of freedom.
  - $\hat{V}(\mathbf{r})$ — the external trapping potential.
- $\hat{H}_0(\mathbf{r}, t)$ has eigenfunctions $|\chi_n(\mathbf{r}, t)\rangle$ with eigenvalues $\varepsilon(\mathbf{r}, t)$.
- Full atomic wave function

$$|\Phi\rangle = \sum_n \Psi_n(\mathbf{r}, t)|\chi_n(\mathbf{r}, t)\rangle.$$

Adiabatic Approximation

Substituting into the Schrödinger equation $i\hbar\partial/\partial t|\Phi\rangle = \hat{H}|\Phi\rangle$ one can write the equation for the coefficients $\Psi_n(\mathbf{r}, t)$ in the form

$$i\hbar\frac{\partial}{\partial t}\Psi = \left[\frac{1}{2M}(-i\hbar\nabla - \mathbf{A})^2 + V + \beta\right]\Psi,$$

where

$$\Psi = \left(\begin{array}{c} \Psi_1 \\ \cdots \\ \Psi_n \end{array}\right),$$

$$\mathbf{A}_{n,n'} = i\hbar\langle\chi_n(\mathbf{r}, t)|\nabla\chi_{n'}(\mathbf{r}, t)\rangle,$$

$$V_{n,n'} = \varepsilon(\mathbf{r}, t)\delta_{n,n'} + \langle\chi_n(\mathbf{r}, t)|\hat{V}(\mathbf{r})|\chi_{n'}(\mathbf{r}, t)\rangle,$$

$$\beta_{n,n'} = -i\hbar\langle\chi_n(\mathbf{r}, t)|\frac{\partial}{\partial t}\chi_{n'}(\mathbf{r}, t)\rangle.$$

Adiabatic Approximation

Non-degenerate states

The first state is well separated from the rest. Off-diagonal terms are neglected.

$$i\hbar \frac{\partial}{\partial t} \Psi_1 = \left[\frac{1}{2M} (-i\hbar\nabla - \mathbf{A})^2 + V + \phi + \beta \right] \Psi_1 ,$$

where

$$\mathbf{A} = \mathbf{A}_{1,1} ,$$
$$V = V_{1,1} ,$$
$$\phi = \frac{1}{2M} \sum_{n \neq 1} \mathbf{A}_{1,n} \cdot \mathbf{A}_{n,1} .$$

Adiabatic Approximation

Degenerate states

The first q dressed states are degenerate and these levels are well separated from the remaining $N - q$

$$i\hbar\frac{\partial}{\partial t}\tilde{\Psi} = \left[\frac{1}{2M}(-i\hbar\nabla - \mathbf{A})^2 + V + \phi + \beta\right]\tilde{\Psi},$$

where $\mathbf{A}$ and V are truncated $q \times q$ matrices,

$$\phi_{n,n'} = \frac{1}{2M}\sum_{m=q+1}^{N} \mathbf{A}_{n,m} \cdot \mathbf{A}_{m,n'}.$$

The effective vector potential $\mathbf{A}$ is called the Mead-Berry connection. The effective scalar potential ϕ is called the Born-Huang potential.

Gauge Transformations

Non-degenerate states

We have freedom of choosing the phase of the adiabatic states

$$|\chi_n(\mathbf{r}, t)\rangle \to e^{-\frac{i}{\hbar} u_n(\mathbf{r},t)} |\chi_n(\mathbf{r}, t)\rangle .$$

The transformation of the potentials

$$\mathbf{A} \to \mathbf{A} + \nabla u_1 ,$$

$$\phi \to \phi - \frac{\partial}{\partial t} u_1 .$$

Gauge Transformations

Degenerate states

The adiabatic basis can be changed by a local unitary transformation $U(\mathbf{r}, t)$

$$\tilde{\Psi} \to U(\mathbf{r}, t)\tilde{\Psi}.$$

The transformation of the potentials

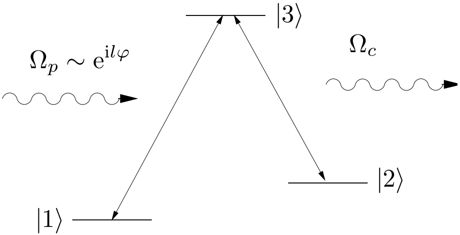
$$\mathbf{A} \to U\mathbf{A}U^\dagger - i\hbar(\nabla U)U^\dagger,$$

$$\phi \to U\phi U^\dagger + i\hbar\frac{\partial U}{\partial t}U^\dagger.$$

The Berry connection **A** is related to a curvature **B** as

$$B_i = \frac{1}{2}\epsilon_{ikl}F_{kl}, \qquad F_{kl} = \partial_k A_l - \partial_l A_k - \frac{i}{\hbar}[A_k, A_l].$$

Λ-type Atoms



Probe beam: $\Omega_p = \mu_{13} E_p$
Control beam: $\Omega_c = \mu_{23} E_c$

Dark state

$$|D\rangle \sim \Omega_c|1\rangle - \Omega_p|2\rangle$$

Destructive interference, cancelation of absorbtion — EIT

Effective Magnetic Field

$$\mathbf{A} = -\hbar \frac{|\zeta|^2}{1 + |\zeta|^2} \nabla S, \qquad \mathbf{B} = \hbar \frac{\nabla S \times \nabla |\zeta|^2}{(1 + |\zeta|^2)^2},$$

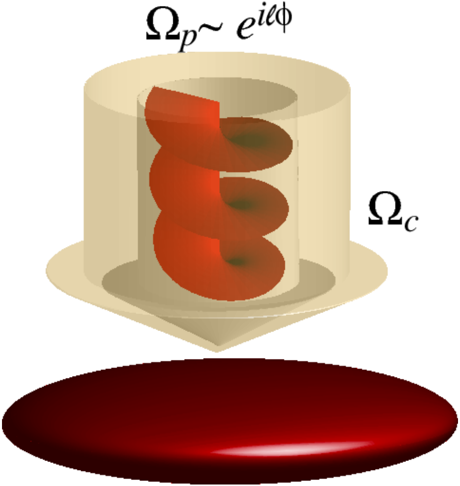
$$\phi = \frac{\hbar^2}{2M} \frac{(\nabla |\zeta|)^2 + |\zeta|^2 (\nabla S)^2}{(1 + |\zeta|^2)^2},$$

where

$$\zeta = \Omega_p / \Omega_c = |\zeta| e^{iS}.$$

- Light beams with relative OAM can introduce an effective magnetic field which acts on the electrically neutral atoms.
- The vector potential **A** is determined by:
  - the gradient of phase difference between the probe and control beams,
  - the ratio between the intensities of the control and probe beams.

Light beams with OAM: Light Vortices



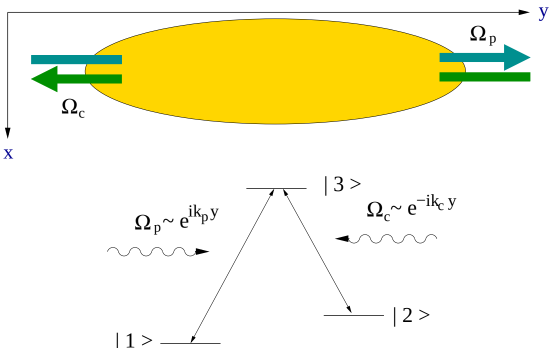
Light vortex

Light vortex — light beam with phase

$$e^{ikz+il\varphi},$$

where φ is azimuthal angle, l — winding number.
Light vortices have orbital angular momentum (OAM) along the propagation axis $M_z = \hbar l$.

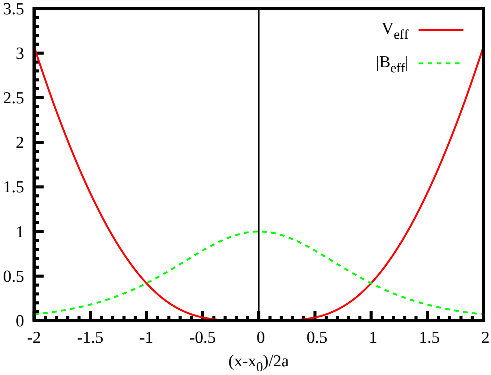
Counterpropagating Light Beams



The phase $S = (k_p + k_c)y$

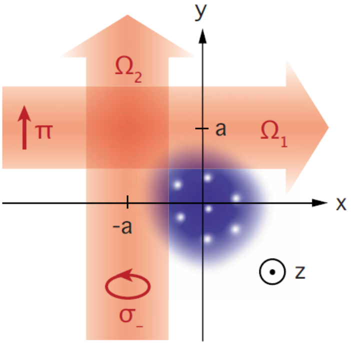
J. Ruseckas, G. Juzeliūnas, P. Öhberg, and M. Fleischhauer, Phys. Rev. A **73** 025602 (2006).

Counterpropagating Gaussian Beams



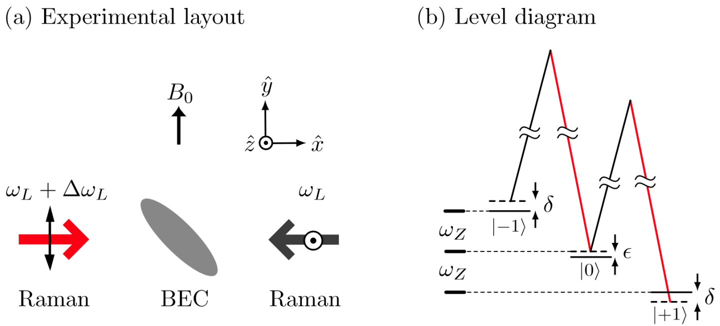
Effective magnetic field B_{eff} and effective trapping potential $V_{\mathrm{eff}} = V + \phi$ produced by counter-propagating Gaussian beams.

Other configurations



K. J. Günter, M. Cheneau, T. Yefsah, S. P. Rath, and J. Dalibard,
Practical scheme for a light-induced gauge field in an atomic Bose gas,
Phys. Rev. A **79**, 011604(R) (2009).

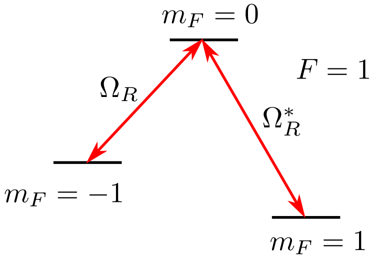
Experimental realisation

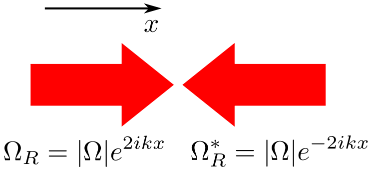


- Counterpropagating σ_+ and π laser beams
- Atom in a real magnetic field (F=1)
- Raman coupling between the ground states $m_F = \pm 1$ and $m_F = 0$.

Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman, Phys. Rev. Lett. **102**, 130401 (2009).

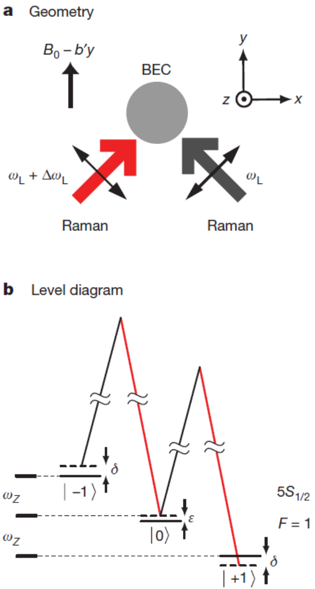
Equivalent to the Λ-type scheme with counterpropagating beams:

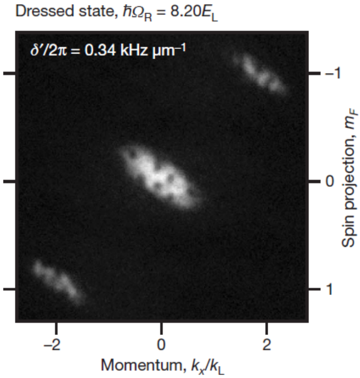




No spatial dependence of the relative amplitude $|\Omega_R / \Omega_R^*| = 1$

Experimental realisation



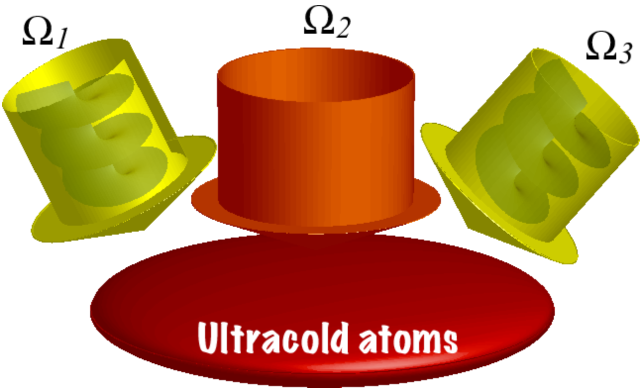


Y.-J. Lin, R. L. Compton,
K. Jiménez-García, J. V. Porto and
I. B. Spielman, Nature, **462**, 628 (2009).

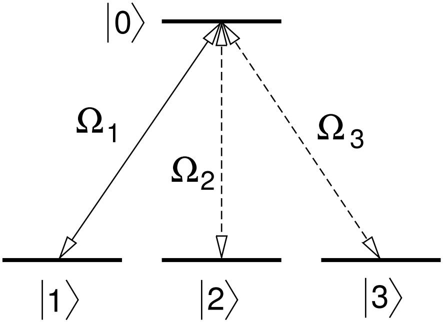
Non-Abelian gauge potentials

- Adiabatic motion of many-level cold atoms in the laser fields varying in space creates effective non-Abelian gauge fields.
- It is possible to simulate motion of the particles in the non-Abelian fields using cold atomic gasses.

Tripod Coupling Scheme



Tripod Coupling Scheme



- Two degenerate dark states
- Non-Abelian gauge potentials

- R. G. Unanyan, M. Fleischhauer, B. E. Shore, and K. Bergmann, Opt. Commun. **155**, 144 (1998).
- J. Ruseckas, G. Juzeliūnas, P. Öhberg, and M. Fleischhauer Phys. Rev. Lett. **95**, 010404 (2005)

Tripod Coupling Scheme

- Two degenerate dark states:

$$|D_1\rangle = \sin\phi e^{iS_{31}}|1\rangle - \cos\phi e^{iS_{32}}|2\rangle,$$
$$|D_2\rangle = \cos\theta\cos\phi e^{iS_{31}}|1\rangle + \cos\theta\sin\phi e^{iS_{32}}|2\rangle - \sin\theta|3\rangle,$$

where

$$\Omega_1 = \Omega\,\sin\theta\,\cos\phi\,e^{iS_1}, \quad \Omega_2 = \Omega\,\sin\theta\,\sin\phi\,e^{iS_2}, \quad \Omega_3 = \Omega\,\cos\theta\,e^{iS_3}.$$

- Vector gauge potential:

$$\mathbf{A}_{11} = \hbar\left(\cos^2\phi\nabla S_{23} + \sin^2\phi\nabla S_{13}\right),$$
$$\mathbf{A}_{12} = \hbar\cos\theta\left(\frac{1}{2}\sin(2\phi)\nabla S_{12} - i\nabla\phi\right),$$
$$\mathbf{A}_{22} = \hbar\cos^2\theta\left(\cos^2\phi\nabla S_{13} + \sin^2\phi\nabla S_{23}\right).$$

Magnetic Monopole

- Laser fields:

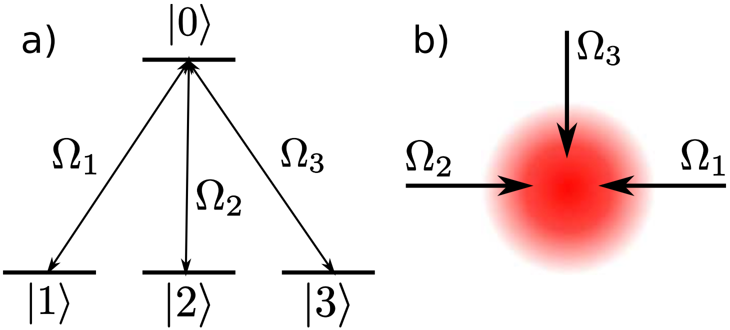
$$\Omega_{1,2} = \Omega_0 \frac{\rho}{R} e^{i(kz \mp \varphi)}, \qquad \Omega_3 = \Omega_0 \frac{z}{R} e^{ik'x}.$$

- The effective magnetic field

$$\mathbf{B} = \frac{\hbar}{r^2} \mathbf{e}_r \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \cdots .$$

- J. Ruseckas, G. Juzeliūnas, P. Öhberg, and M. Fleischhauer, Phys. Rev. Lett. **95**, 010404 (2004).
- V. Pietila, M. Mottonen, Phys. Rev. Lett. **102**, 080403 (2009).
- V. Pietila, M. Mottonen, Phys. Rev. Lett. **103**, 030401 (2009).

Ultrarelativistic Dirac fermions



$$\Omega_1 = \Omega \sin\theta e^{-i\kappa x}/\sqrt{2}\,, \quad \Omega_2 = \Omega \sin\theta e^{i\kappa x}/\sqrt{2}\,, \quad \Omega_3 = \Omega \cos\theta e^{-i\kappa y}$$

where

$$\theta = \theta_0\,, \qquad \cos\theta_0 = \sqrt{2} - 1$$

Ultrarelativistic Dirac fermions

The Hamiltonian

$$H_{\mathbf{k}} = \frac{\hbar^2}{2m}(\mathbf{k} + \kappa'\sigma_\perp)^2 + V_1$$

with

$$\kappa' = \kappa \cos\theta_0\,, \qquad \sigma_\perp = \mathbf{e}_x\sigma_x + \mathbf{e}_y\sigma_y$$

For small wave vectors $k \ll \kappa'$, the atomic Hamiltonian reduces to the Hamiltonian for the 2D relativistic motion of a two-component massless particle of the Dirac type known also as the Weyl equation

$$H_{\mathbf{k}} = \hbar v_0 \mathbf{k} \cdot \sigma_\perp + V_1 + m v_0^2$$

where the velocity $v_0 = \hbar\kappa'/m$ correspons to the velocity of ligth. For cold atoms this velocity is of the order 1 $\mathrm{cm/s}$.

Ultrarelativistic Dirac fermions

The Hamiltonian $H_{\mathbf{k}}$ commutes with the 2D chirality operator

$$\sigma_{\mathbf{k}} = \mathbf{k} \cdot \sigma_{\perp} / k$$

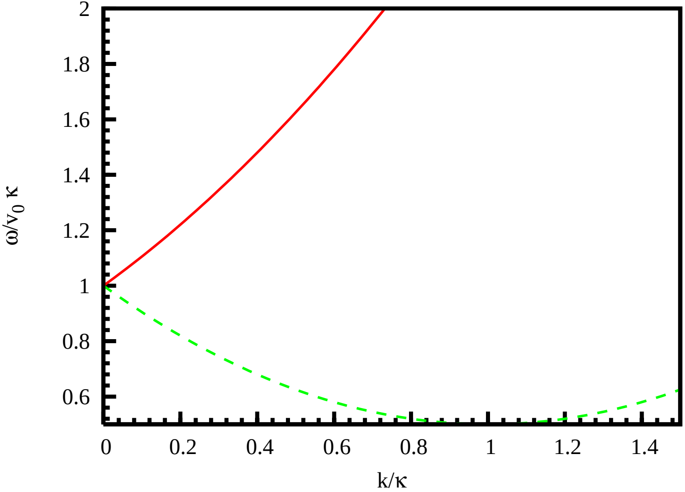
The dispersion

$$\hbar \omega_{\mathbf{k}}^{\pm} = \hbar v_0 (k^2 / 2\kappa' \pm k) + V_1 + m v_0^2$$

For small wave vectors

$$\hbar \omega_{\mathbf{k}}^{\pm} = \pm \hbar v_0 k + V_1 + m v_0^2$$

Ultrarelativistic Dirac fermions



Zitterbewegung

The Hamiltonian for small momenta with an additional scalar potential:

$$H = v_0 \sigma_\perp \cdot \mathbf{p} + V \sigma_z$$

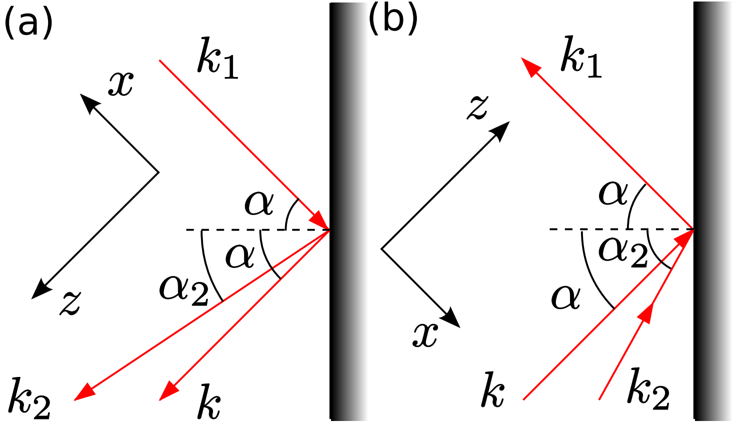
The velocity operator

$$\mathbf{v} \equiv \dot{\mathbf{r}} = \frac{1}{i\hbar}[\mathbf{r}, H] = v_0 \sigma_\perp$$

The eigenfunctions of the Hamiltonian do not have a definite velocity.
Consequence: oscillations in the movement of the wave packet.

- J. Y. Vaishnav and C. W. Clark, Phys. Rev. Lett. **100**, 153002 (2008).
- M. Merkl, F. E. Zimmer, G. Juzeliūnas, and P. Öhberg, Europhys. Lett. **83**, 54002 (2008).

Negative reflection



Negative reflection

Angle of the negative reflection

$$\alpha_2 = \arcsin\left(\frac{k}{k_2}\sin\alpha\right)$$

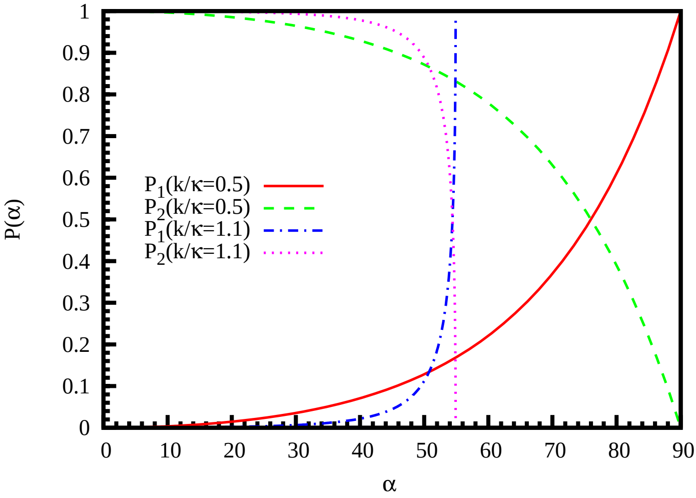
where $k_2 = 2\kappa - k$. Reflection coefficients

$$r_1 = \frac{e^{i\alpha} - e^{i\alpha_2}}{e^{-i\alpha} + e^{i\alpha_2}}\,, \qquad r_2 = -1 - r_1\,.$$

The corresponding reflection probabilities

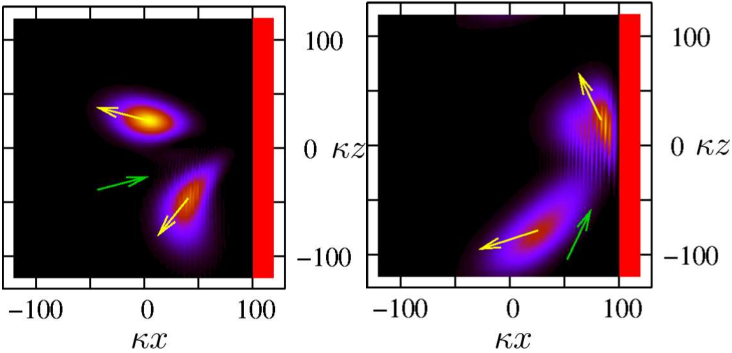
$$P_1 = |r_1|^2\,, \qquad P_2 = \frac{\cos\alpha_2}{\cos\alpha}|r_2|^2$$

Negative reflection



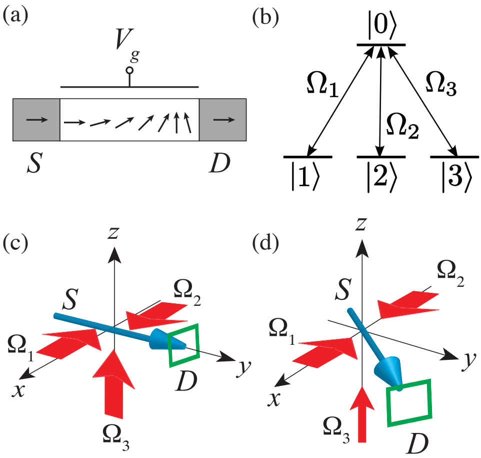
Reflection probabilities.

Negative reflection



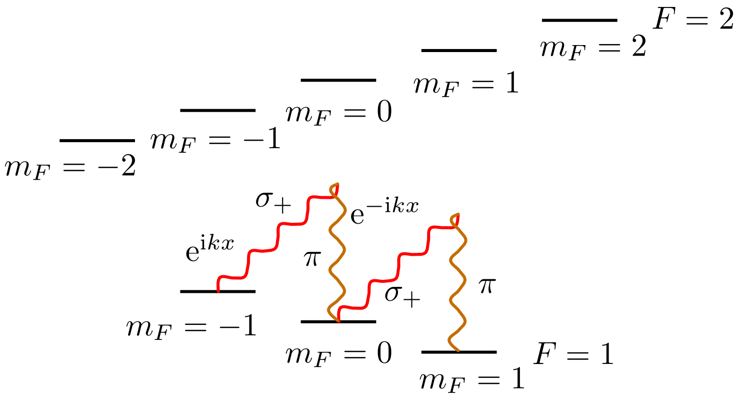
G. Juzeliūnas, J. Ruseckas, A. Jacob, L. Santos, and P. Öhberg, Phys. Rev. Lett. **100**, 200405 (2008).

Spin field effect transistor with ultracold atoms



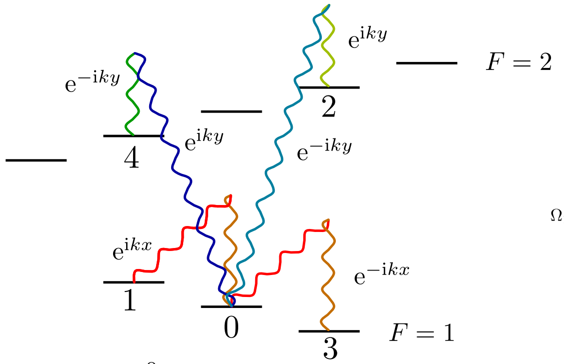
J. Y. Vaishnav, J. Ruseckas, C. W. Clark, and G. Juzeliūnas, Phys. Rev. Lett. **101**, 265302 (2008).

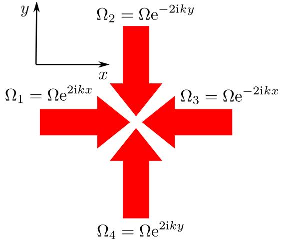
Tetrapod scheme with counter-propagating beams

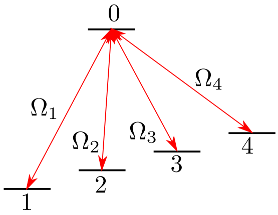


Lambda scheme, no Raman coupling to the $F = 2$ levels

Tetrapod scheme with counter-propagating beams







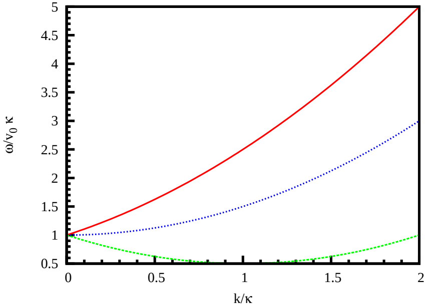
Raman coupling to the $F = 2$ levels

Tetrapod scheme with counter-propagating beams

Spin-1 Rashba-type Hamiltonian

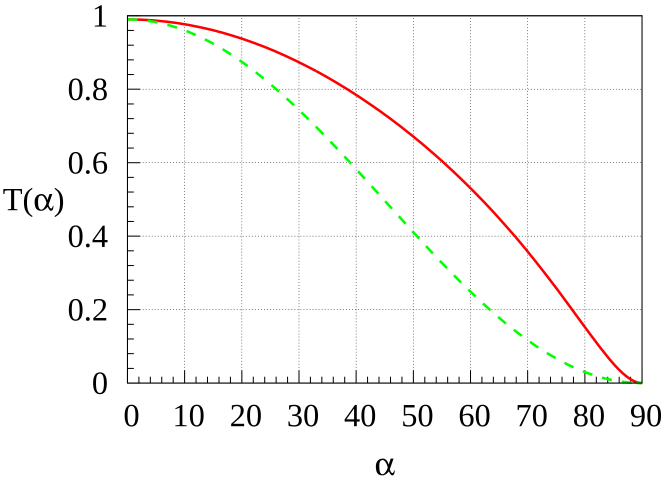
$$\hat{H} = \frac{1}{2m}(\hat{\mathbf{p}} + \hbar\kappa\mathbf{J}_{\perp})^2 + V$$

where $\mathbf{J}_{\perp}$ is the projection of spin-1 operator onto the xy plane.



Dispersion

Comparison of transmission probabilities for spin-1/2 and spin-1 systems



Summary

- Light beams with relative orbital angular momentum can introduce Abelian and non-Abelian effective gauge potentials acting on the electrically neutral atoms.
- Non-Abelian fields can be formed for cold atoms using the plane-wave seups. This was not possible for the Abelian fields.
- Atomic motion in non-Abelian fields exhibit a number of non-trivial features, such as their quasirelativistic behaviour or the negative refraction and reflection.
- The plane wave setups can lead to the spin $1/2$ or the spin 1 Rashba-type Hamiltonian for cold atoms.

Thank you!