Spin-orbit coupling for ultracold atoms and for slow light

Julius Ruseckas

Institute of Theoretical Physics and Astronomy, Vilnius University, Lithuania

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Outline

Spin-orbit coupling

- Linear in momentum SOC ($\mathbf{p} \cdot \boldsymbol{\sigma}$ type term) has been widely studied in condensed matter physics
- SOC leads to new phenomena:
  - topological insulators
  - quantum anomalous Hall effect
  - topological superconductors

Synthetic spin-orbit coupling

- Synthetic SOC can be created for neutral atoms
- SOC significantly enriches the system
- For ultracold atoms synthetic SOC leads to:
    - stripe phase and vortex structure in the ground states of spin-orbit-coupled Bose-Einstein condensates
    - Rashba pairing bound states
    - topological superfluidity in fermionic gases
    - superfluidity and Mott-insulating phases of spin-orbit-coupled quantum gases in optical lattice

Synthetic spin-orbit coupling

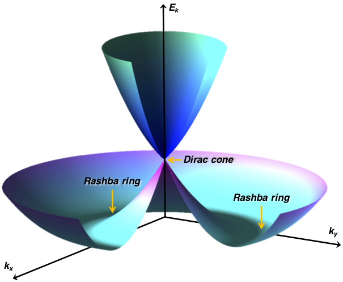
Currently a great deal of interest in SOC for ultracold atoms

Reviews on SOC in quantum gases:

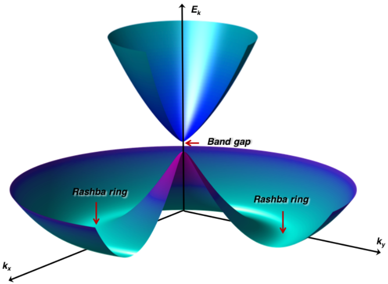
- J. Dalibard, F. Gerbier, G. Juzeliūnas and P. Öhberg, Rev. Mod. Phys. **83**, 1523 (2011).
- V. Galitski and I. B. Spielman, Nature **494**, 49 (2013).
- N. Goldman, G. Juzeliūnas, P. Öhberg and I. B. Spielman, Rep. Progr. Phys. **77**, 126401 (2014).
- H. Zhai, Rep. Progr. Phys. **78** 026001 (2015).

Dispersion of centre of mass motion for a particle affected by Rashba SOC

$$H = \frac{\mathbf{p}^2}{2m} + \frac{1}{m}\mathbf{p}\cdot\mathbf{A} + b\sigma_z\,, \qquad \mathbf{A} = \chi(\sigma_x\mathbf{e}_x + \sigma_y\mathbf{e}_y)$$



Zero Zeeman term, $b = 0$



Non-zero Zeeman term, $b \neq 0$

Spin-orbit coupling for ultracold atoms

- Initial proposal: atoms moving in electric field

  M. Ericsson and E. Sjöqvist Phys. Rev. A **65**, 013607 (2001).

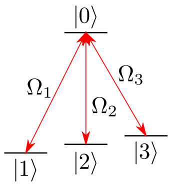
  Relativistic effect: extremely weak for ultracold atoms
- Using optical lattices

  A. M. Dudarev, R. B. Diener, I. Carusotto, and Q. Niu Phys. Rev. Lett. **92** 153005 (2004).

  Z. Wu *et al*, arXiv:1511.08170 (cond-mat.quant-gas) (2015).
- Using light-induced geometric potentials for adiabatic motion of atoms in a pair of degenerate internal states

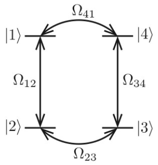
Spin-orbit coupling for ultracold atoms

- Tripod configuration
  - J. Ruseckas, G. Juzeliūnas, P. Öhberg, and M. Fleischhauer, Phys. Rev. Lett. **95**, 010404 (2005).
  - T. D. Stanescu and V. Galitski, Phys. Rev. B **75** 125307 (2007).
  - G. Juzeliūnas, J. Ruseckas, M. Lindberg, L. Santos, and P. Öhberg, Phys. Rev. A **77**, 011802(R) (2008).



- Ring coupling scheme

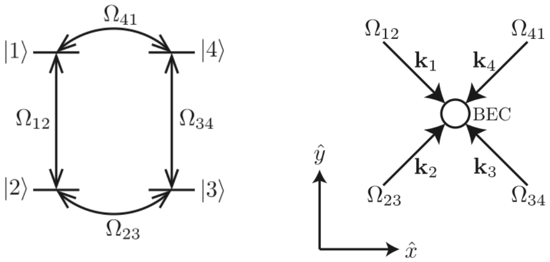
  D. L. Campbell, G. Juzeliūnas and I. B. Spielman, Phys. Rev. A **84**, 025602 (2011).



Ring coupling setup

D. L. Campbell, G. Juzeliūnas and I. B. Spielman, Phys. Rev. A **84**, 025602 (2011).

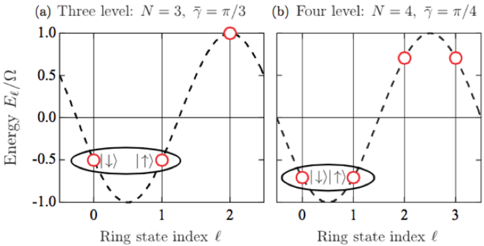
N atomic internal states coupled in a cyclic manner with a cyclic phase $\bar{\phi} = \sum_j \phi_j = \pi$



Laser fields represent counter-propagating plane waves,
$\Omega_{j,j+1} = \Omega \mathrm{e}^{\mathrm{i}\mathbf{k}_j \cdot \mathbf{r} + \mathrm{i}\phi_j}$

Ring coupling setup

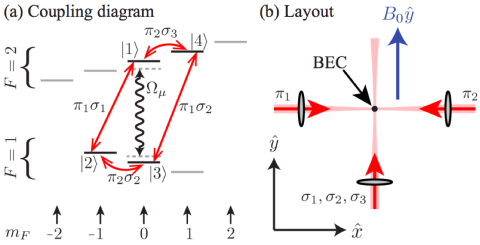
Double degenerate ground state for $N = 3$ and $N = 4$ if $\bar{\phi} = \pi$



- 2D SOC for cold atoms in ground-state manifold
- Zeeman term if $\bar{\phi} \neq \pi$

Ring coupling setup

Possible implementation of the ring coupling setup using the Raman transitions, $N = 4$

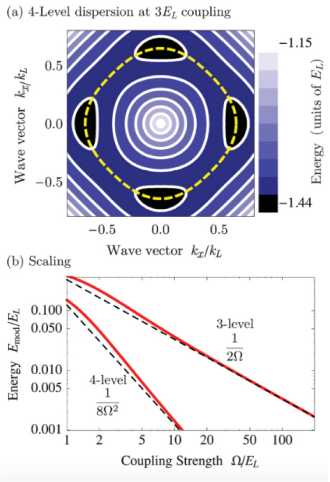


- Involves $F = 2$ manifold: shorter lifetimes.
- Phase sensitive scheme.

Ring coupling setup

Scheme with $N = 4$

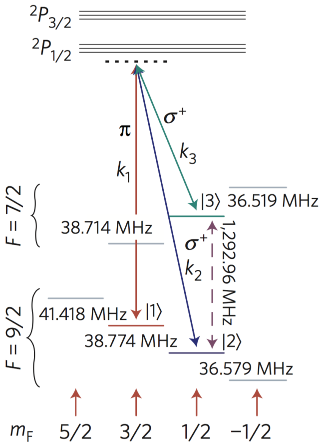
- dispersion better coverges to the Rashba ring compared to $N = 3$
- is better suited to observe the Rashba ring



Ring coupling setup, experiment for $N = 3$

L. Huang *et al*, Nature Phys. **12**, 540 (2016).

- Far detuned tripod, $N = 3$
- $|1\rangle$, $|2\rangle$ and $|3\rangle$ are coupled in a cyclic manner via virtual excited states
- Involves different F manifolds
- Dirac cone observed, not Rashba ring



Spin-orbit coupling for ultracold atoms

Schemes for creating 2D SOC

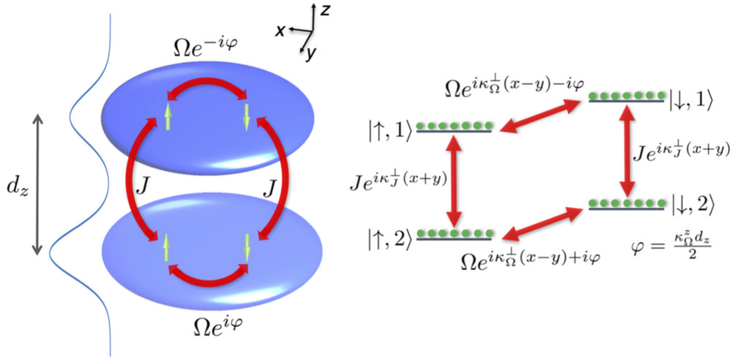
- involve complex atom-light coupling
- with many atomic states,
- have been only very recently implemented experimentally

Our proposal

Rashba-type SOC using only two atomic internal states in a bilayer system

S.-W. Su *et al*, Phys. Rev. A **93**, 053630 (2016).

SOC using a bilayer system

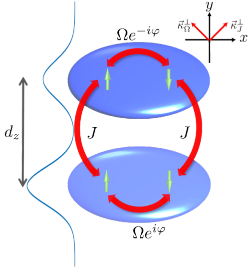


Setup:

- two atomic internal states $|\uparrow\rangle$ and $|\downarrow\rangle$
- double well potential
  leading to an extra layer index 1 and 2

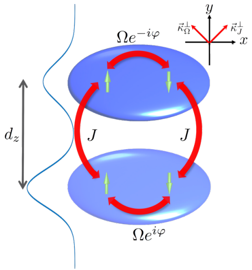
SOC using a bilayer system

- Double well potential provides an extra degree of freedom by the layer index
- Four combined spin-layer states $|\mathrm{spin}, \mathrm{layer}\rangle$ ($\mathrm{spin} = \uparrow, \downarrow$, $\mathrm{layer} = 1, 2$) serve as the atomic states in the $N = 4$ ring coupling scheme



SOC using a bilayer system

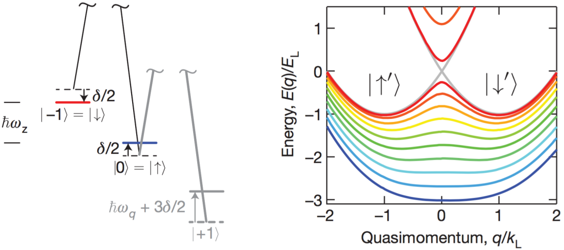
- J — interlayer laser-assisted tunneling (recoil along $x+y$)
- Ω — intralayer Raman coupling (recoil along $x-y$) with $2\varphi=\pi$ phase shift between the layers
  $\varphi=\kappa_\Omega^z d_z/2$, where κ_Ω^z is the Raman recoil along z
- Zeeman term if $2\varphi\neq\pi$



SOC using a bilayer system

- 1D SOC in each layer due to the Raman recoil in the same direction $x - y$
- Similar to

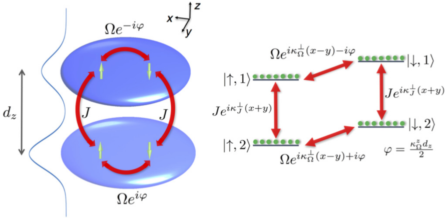
  Y.-J. Lin, K. Jiménez-García and I. B. Spielman, Nature **471**, 83 (2011).



$$H = \frac{1}{2m}(p - \chi\sigma_y)^2 + \frac{\Omega}{2}\sigma_z$$

- 1D SOC and interlayer tunneling $\rightarrow$ Rashba SOC

Single particle Hamiltonian



$$\hat{H}_0 = \hat{H}_{\text{atom}} + \hat{H}_{\text{intra}} + \hat{H}_{\text{inter}} + \hat{H}_{\text{extra}}$$

$$\hat{H}_{\text{atom}} = \int \mathrm{d}^2\mathbf{r}_\perp \sum_{j,\gamma} \hat{\psi}^\dagger_{\gamma j} \frac{\hbar^2 \mathbf{k}_\perp^2}{2m} \hat{\psi}_{\gamma j}$$

$$\hat{H}_{\text{intra}} = \int \mathrm{d}^2\mathbf{r}_\perp \Omega \left[\mathrm{e}^{\mathrm{i}\varphi} \hat{\psi}^\dagger_{\uparrow 1} \hat{\psi}_{\downarrow 1} + \mathrm{e}^{-\mathrm{i}\varphi} \hat{\psi}^\dagger_{\uparrow 2} \hat{\psi}_{\downarrow 2} + \mathrm{H.c.} \right]$$

$$\hat{H}_{\text{inter}} = \int \mathrm{d}^2\mathbf{r}_\perp \sum_{\gamma} J \hat{\psi}^\dagger_{\gamma 2} \hat{\psi}_{\gamma 1} + \mathrm{H.c.}$$

An extra term due to recoil: yields SOC

$$\hat{H}_{\text{extra}} = \int \mathrm{d}^2\mathbf{r}_\perp \frac{\hbar^2 \kappa}{m} \left[\hat{\psi}^\dagger_{\uparrow 2} k_x \hat{\psi}_{\uparrow 2} - \hat{\psi}^\dagger_{\downarrow 1} k_x \hat{\psi}_{1\downarrow} + \hat{\psi}^\dagger_{\downarrow 2} k_y \hat{\psi}_{\downarrow 2} - \hat{\psi}^\dagger_{\uparrow 1} k_y \hat{\psi}_{\uparrow 1} \right]$$

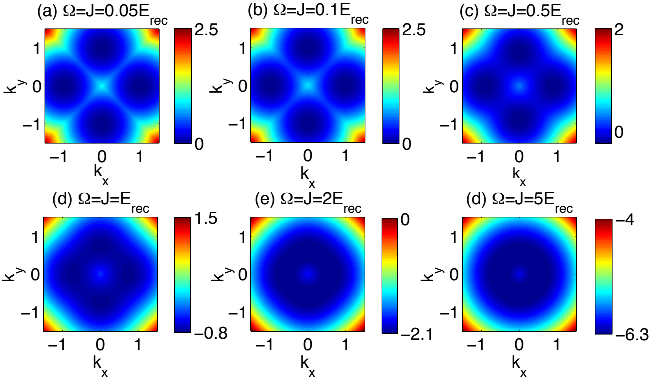
Single particle dispersion

Four dispersion branches

$$E_{s_1,s_2} = 1 + k^2 + s_1\sqrt{\Omega^2 + J^2 + 2k^2 + 2s_2 a_{\mathbf{k}}}\,, \qquad s_1, s_2 = \pm 1$$

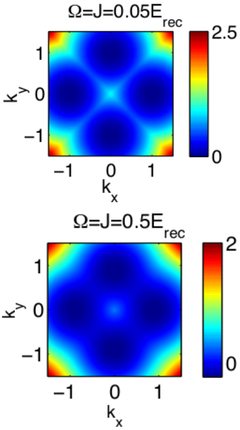
$$a_{\mathbf{k}} = \sqrt{\Omega^2(k_x + k_y)^2 + J^2(k_x - k_y)^2 + (k_x^2 - k_y^2)^2}$$

Lower dispersion branch at various $\Omega = J$:



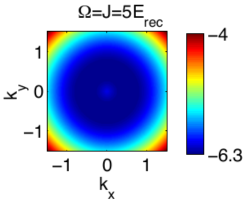
Single particle ground state

- Weak coupling, $\Omega/E_{\rm rec} = J/E_{\rm rec} \ll 1$
  - Four minima at $\mathbf{k} = (\pm\kappa, 0), (0, \pm\kappa)$
  - Ground state at the minima contains one spin-layer component
- Moderate coupling, $\Omega/E_{\rm rec} = J/E_{\rm rec} \lesssim 1$
  - Four minima at $\mathbf{k} = (\pm\kappa, 0), (0, \pm\kappa)$
  - Ground state at the minima contains three spin-layer components



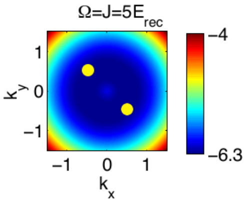
Single particle ground state

- Strong coupling, $\Omega/E_{\rm rec} = J/E_{\rm rec} \gg 1$
  - Degenerate ring minimum $\mathbf{k} = \frac{\kappa}{2}(\cos\phi_{\mathbf{k}}, \sin\phi_{\mathbf{k}})$
  - Ground state at the minimum contains four spin-layer components
  - Population of each spin-layer component depends on $\phi_{\mathbf{k}}$
  - Population difference between the two layers oscillates with $\phi_{\mathbf{k}}$



$$\Delta\rho = (|\psi_{\uparrow,1}|^2+|\psi_{\downarrow,1}|^2)-(|\psi_{\uparrow,2}|^2+|\psi_{\downarrow,2}|^2) = \sqrt{2}(\sin\phi_{\mathbf{k}}+\cos\phi_{\mathbf{k}})$$

$\Delta\rho = 0$ for $\phi_{\mathbf{k}} = 3\pi/4$ and $\phi_{\mathbf{k}} = 7\pi/4$



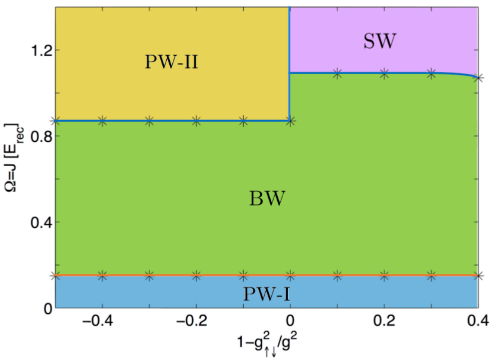
Many-body ground state

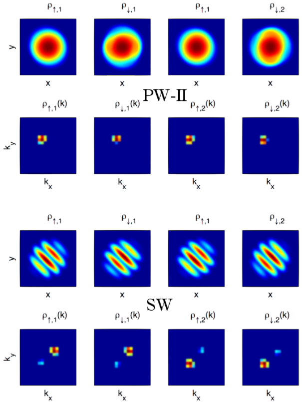
Atom-atom interaction — only within the layers
Full Hamiltonian:

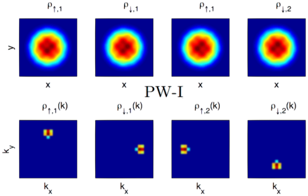
$$\hat{H} = \hat{H}_0 + \hat{H}_{\rm int}$$

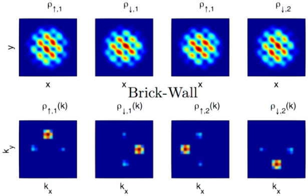
$$\hat{H}_{\rm int} = \int \mathrm{d}^2\mathbf{r}_\perp \sum_{j=1,2} \left(\frac{g_\uparrow}{2}\hat{n}_{\uparrow j}^2 + \frac{g_\downarrow}{2}\hat{n}_{\downarrow j}^2 + g_{\uparrow\downarrow}\hat{n}_{\uparrow j}\hat{n}_{\downarrow j} \right)$$

Many-body ground state phase diagram





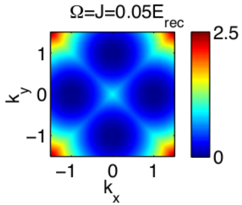


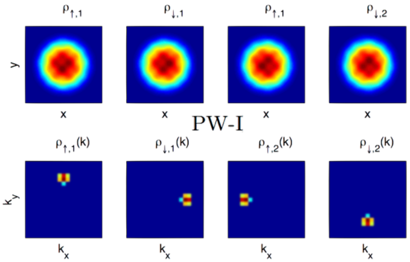


Many-body ground state

Weak coupling, $\Omega/E_{\mathrm{rec}} = J/E_{\mathrm{rec}} \ll 1$

- each component of the ground state carries single plane-wave factor — plane-wave phase
- zero intralayer spin polarization (equal population of spin-up and spin-down in each layer)

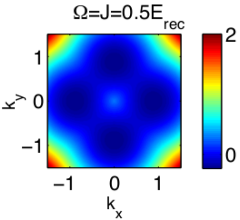


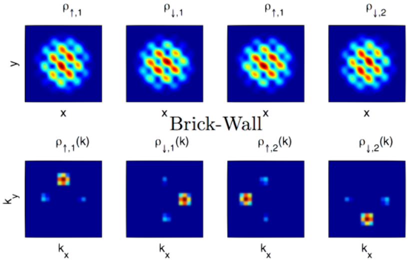


Many-body ground state

Moderate coupling, $\Omega/E_{\rm rec} = J/E_{\rm rec} \lesssim 1$

- each component contains three plane-wave contributions — brick wall phase
- zero intralayer spin polarization

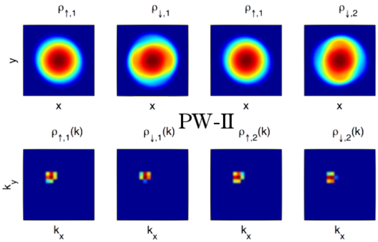




Many-body ground state

Strong coupling, $\Omega/E_{\rm rec} = J/E_{\rm rec} \gg 1$

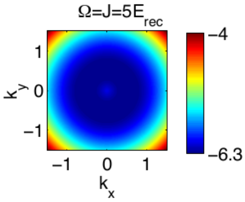
- $g_{\uparrow\downarrow}^2 > g_\uparrow g_\downarrow$
- Spin-independent interaction energy is minimized for $\Delta\rho = 0$
- This gives $\phi_{\mathbf{k}} = 3\pi/4$ or $\phi_{\mathbf{k}} = 7\pi/4$: along diagonal $x - y$, direction of Raman recoil
- Interaction-induced anisotropy for bilayer system
- BEC forms at one of these two momentum points (PW-II phase)
  Usually: BEC with no preferred momentum on the Rashba ring;
- Nonzero intralayer spin polarization

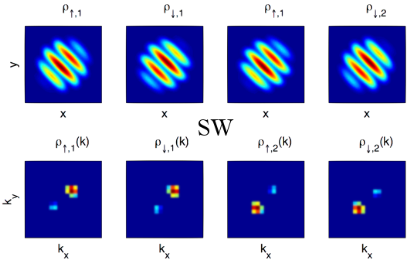


Many-body ground state

Strong coupling, $\Omega/E_{\mathrm{rec}} = J/E_{\mathrm{rec}} \gg 1$

- $g_{\uparrow\downarrow}^2 < g_\uparrow g_\downarrow$
- superposition of two plane waves with momenta $\mathbf{k} = \frac{\kappa}{2}(\cos\frac{\pi}{4}, \sin\frac{\pi}{4})$ and $\mathbf{k} = \frac{\kappa}{2}(\cos\frac{5\pi}{4}, \sin\frac{5\pi}{4})$
- standing wave phase
- zero intralayer spin polarization

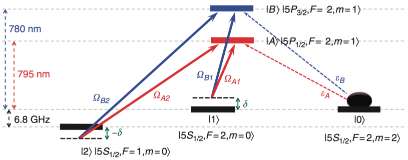


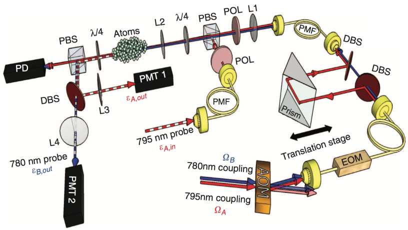


Spin-orbit coupling for slow light?

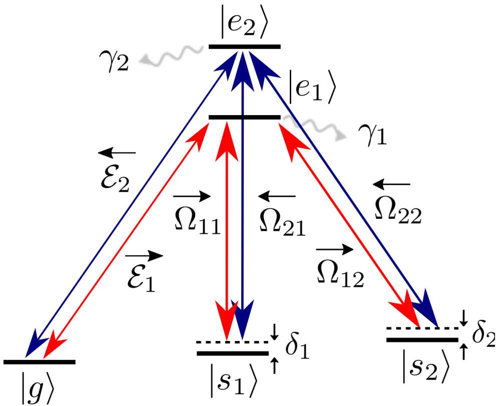
Spinor slow light

M.-J. Lee, J. Ruseckas, *et al*, Nat. Commun. **5**, 5542 (2014).



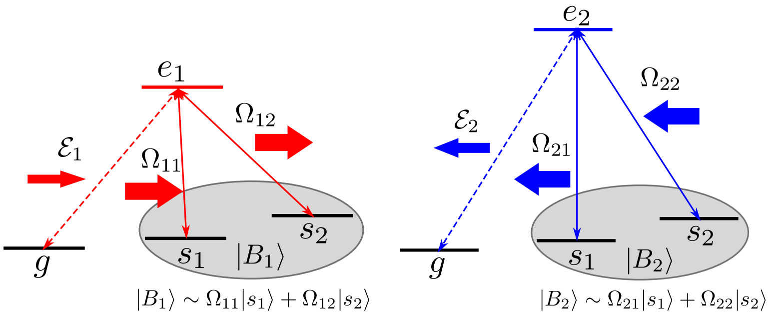


Double tripod setup



- R. G. Unanyan, J. Otterbach, M. Fleischhauer, J. Ruseckas, V. Kudriašov, G. Juzeliūnas, Phys. Rev. Lett. **105**, 173603 (2010).
- J. Ruseckas, V. Kudriašov, G. Juzeliūnas, R. G. Unanyan, J. Otterbach, M. Fleischhauer, Phys. Rev. A **83**, 063811 (2011).

Double tripod setup

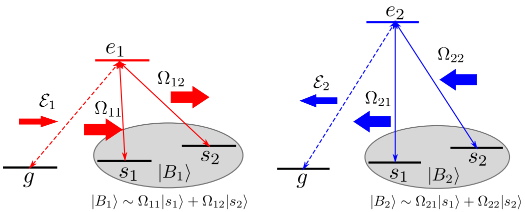


Probe fields $\mathcal{E}_1$ and $\mathcal{E}_2$ are coupled via atomic coherences if $\langle B_1|B_2\rangle \neq 0$

Double tripod setup

Limiting cases:

- $\langle B_1 | B_2 \rangle = 0$ — two not connected Λ schemes
- $\langle B_1 | B_2 \rangle = 1$ — double Λ setup
- $0 < |\langle B_1 | B_2 \rangle| < 1$ — two connected Λ schemes



Propagation of slow light

Matrix representation — Spinor slow light:

$$\mathcal{E} = \begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \end{pmatrix}, \qquad \hat{\Omega} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}, \qquad \hat{\delta} = \begin{pmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{pmatrix}$$

δ_1 and δ_2 are the detunings from two-photon resonance.

Equation for two-component probe field in the atomic cloud:

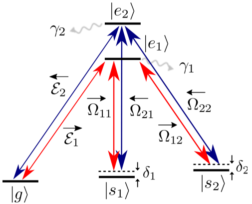
$$(c^{-1} + \hat{v}^{-1})\frac{\partial}{\partial t}\mathcal{E} + \frac{\partial}{\partial z}\mathcal{E} + \mathrm{i}\hat{v}^{-1}\hat{D}\mathcal{E} = 0$$

Similar to the equation for probe field in Λ scheme, only with matrices.

$\hat{D} = \hat{\Omega}\hat{\delta}\hat{\Omega}^{-1}$ is a matrix due to two-photon detuning,

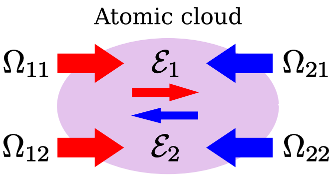
$$\hat{v}^{-1} = \frac{g^2 n}{c}(\hat{\Omega}^{\dagger})^{-1}\hat{\Omega}^{-1}$$

is a matrix of inverse group velocity (not necessarily diagonal).



Spinor slow light

- The group velocity is a non-diagonal matrix
- Individual probe fields do not have a definite group velocity
- Only special combinations of both probe fields (normal modes) propagate in the atomic cloud with the definite (and different) velocities
- This difference in velocities causes interference between probe fields

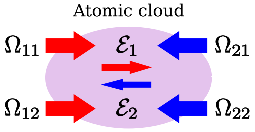


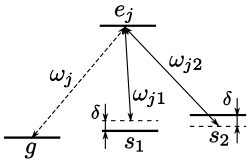
Spin-orbit coupling for slow light

- Counter-propagating beams in double tripod setup
- Non-zero two photon detuning
  $\delta_1 = -\delta_2 \equiv \delta \neq 0$
- Diagonal matrix of group velocity
- Dirac type equation with non-zero mass for two component slow light:

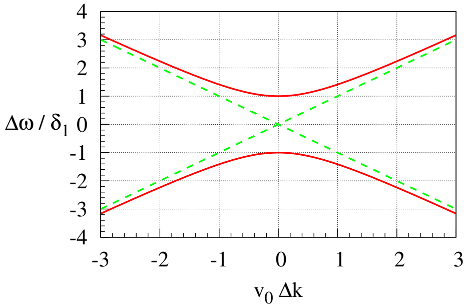
$$\mathrm{i}\frac{\partial}{\partial t}\tilde{\mathcal{E}} = -\mathrm{i}v_0\sigma_z\frac{\partial}{\partial z}\tilde{\mathcal{E}} + \delta\sigma_y\tilde{\mathcal{E}}$$

  Here $v_0 = \frac{c\Omega^2}{g^2 n}$
- A gap in dispersion
  ("electron-positron" type spectrum)



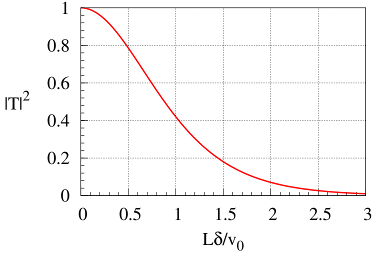


Photonic band-gap for two-component slow light



- Relativistic particle-antiparticle dispersion: $\Delta\omega^{\pm} = \pm\sqrt{v_0^2 \Delta k^2 + \delta^2}$
- $\hbar\delta = m v_0^2$ — gap width, m — polariton effective mass

Dirac equation for two-component slow light



- Reflection and transmission coefficients at the gap center ($\Delta\omega = 0$):

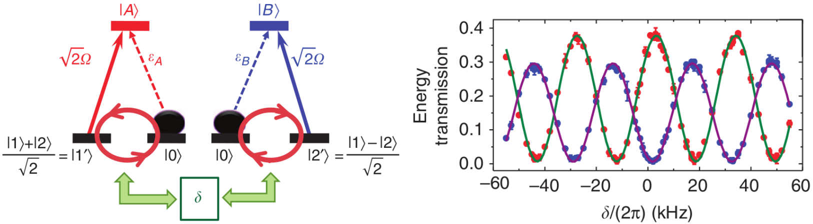
$$T = \cosh^{-1}(L/\lambda_{\mathrm{C}})\,, \quad R = \tanh(L/\lambda_{\mathrm{C}})$$

- $\lambda_{\mathrm{C}} = \hbar/mv_0 = v_0/\delta$ — Compton wave-length of the polariton.
- The Compton wave-length determines the polariton tunneling length.

Spinor slow light for co-propagating beams

Two-photon detuning causes oscillations in the intensities of transmitted probe fields

M.-J. Lee, J. Ruseckas, *et al*, Nat. Commun. **5**, 5542 (2014).



- Detuning can be caused by the interaction
- For example: interaction between Rydberg atoms → generation of squeezed slow light due to atom-atom interaction

  J. Ruseckas, I. A. Yu, G. Juzeliūnas, arXiv:1606.00562 (quant-ph)

Summary

- Spin-orbit coupling for ultracold atoms can be created using the light induced non-Abelian vector potential
- A new scheme: Rashba type SOC using a bilayer system
  - Only two atomic internal states are needed
  - Various BEC many-body phases
  - Bilayer system for atomic fermions, arXiv:1603.06698 (cond-mat.quant-gas)
- Spinor slow light can be created using double-tripod level scheme
- Under certain conditions propagation of spinor slow light is described by 1D Dirac-type equation with spin-orbit coupling

Thank you for your attention!

SOC using a bilayer system

Setup: three laser beams:

- $\mathbf{E}_0$, $\mathbf{E}_1$ induce interlayer tunneling with recoil along $x + y$
- $\mathbf{E}_0$, $\mathbf{E}_2$ produce Raman transitions in each layer with recoil along $x - y$
- $\mathbf{E}_0$ — circularly polarized light
- $\mathbf{E}_1$, $\mathbf{E}_2$ — linearly polarized light