Journal Club:
Critical wind speed at which trees break

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Critical wind speed at which trees break

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Data from storms suggest that the critical wind speed at which trees break is constant ($\simeq$42 m/s), regardless of tree characteristics. We question the physical origin of this observation both experimentally and theoretically. By combining Hooke's law, Griffith's criterion, and tree allometry, we show that the critical wind speed indeed hardly depends on the height, diameter, and elastic properties of trees.

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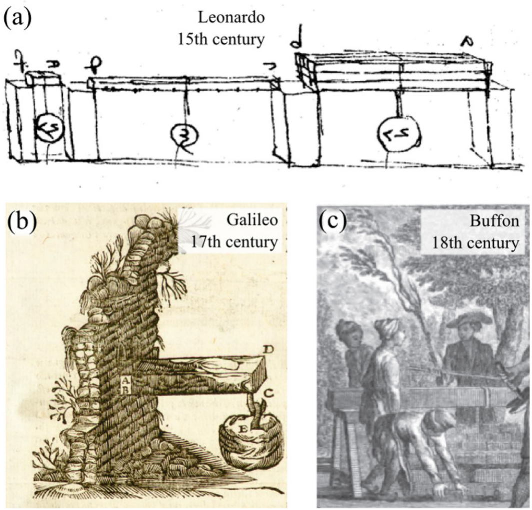
I. INTRODUCTION

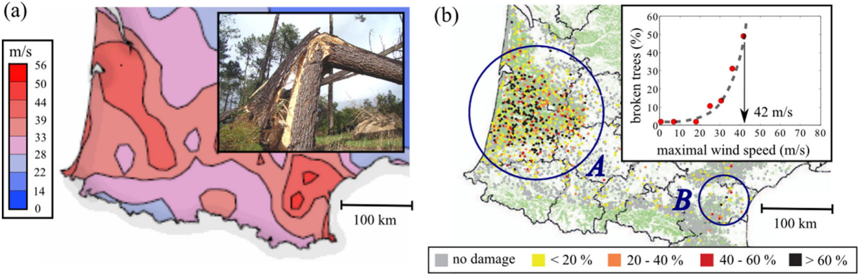
Since the occurrence and severity of storms will probably increase in the coming decades [1–3], the modeling of wind impact on trees deserves special attention [4–11]. The resistance of wood has been a concern for a long time, mainly for human constructions [12]. Seminal works on this subject are briefly presented in Fig. 1. Leonardo first studied the resistance of human constructions and did some preliminary rupture tests on wood beams [13,14] [Fig. 1(a)]. His conclusions made for breaks [stem lodging, as shown in the inset of Fig. 2(a)] [28]. Both the trunk and the root system are under stress, and failure occurs at the weakest part of the tree. During storm Klaus, both kinds of lodging were reported [26], with six million cubic meters of wood due to trunk breakage. Our study focuses on the limit of strong roots, so that the vulnerability of trees arises from the breakage of the trunk. Our objective will be to exhibit the minimal ingredients to describe the critical wind speed causing trunk breakage.

Broken trees

Early studies on the resistance of wood

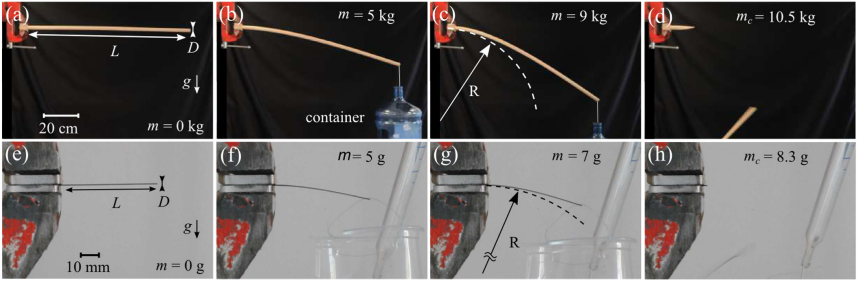


Results of a storm

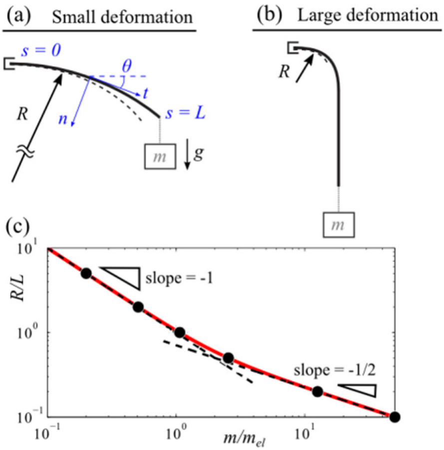


The storm Klaus in France, January 24th, 2009

Experiment I



How to measure critical curvature?



If the rod is weakly bent ($R \gg L$), the balance of the bending moment

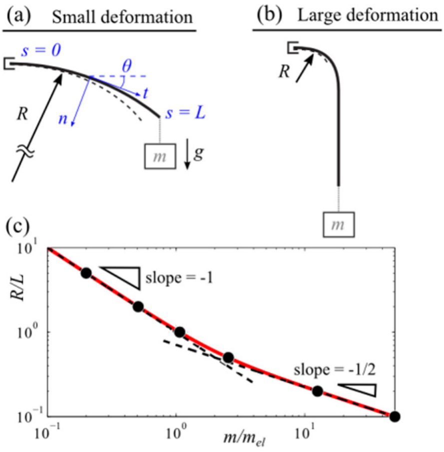
$$mgL \sim \frac{EI}{R}$$

Here

- E is the elastic modulus
- I is the moment of inertia of the rod cross section.

$$I = \frac{\pi}{64} D^4$$

How to measure critical curvature?



Then

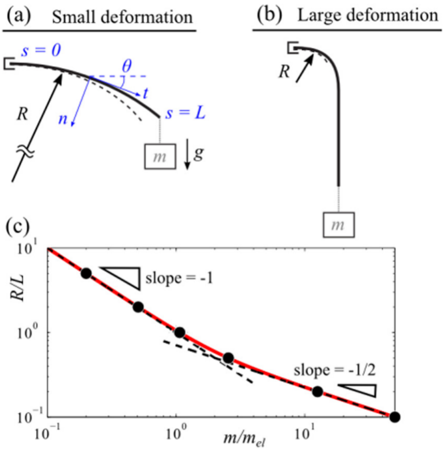
$$\frac{R}{L} \sim \frac{m_{\mathrm{el}}}{m}$$

where

$$m_{\mathrm{el}} = \frac{EI}{gL^2}$$

is the characteristic mass when $R \sim L$.

How to measure critical curvature?



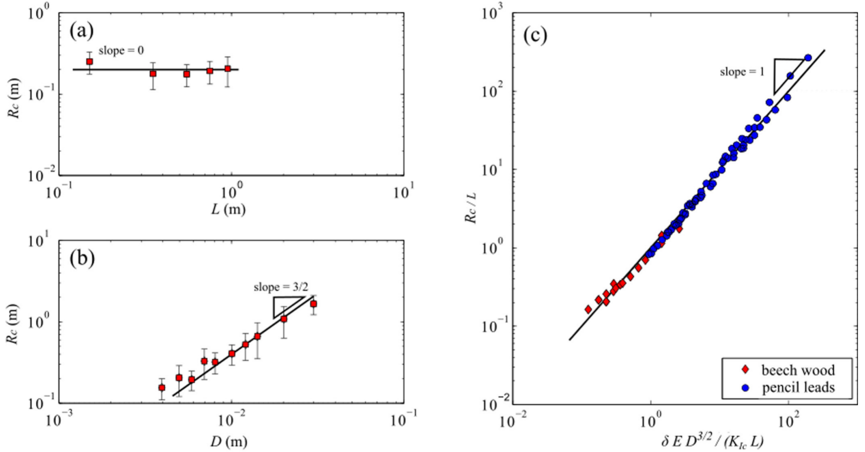
If the rod is strongly bent ($R \ll L$), the characteristic lever arm becomes R and the torgue balance is

$$mgR \sim \frac{EI}{R}$$

therefore

$$\frac{R}{L} \sim \sqrt{\frac{m_{\mathrm{el}}}{m}}$$

Experimental results



Critical curvature

Critical value of strain is related to the critical value of stress σ_c

$$\sigma_c = E\epsilon_c$$

Since strain is related to the curvature as

$$\epsilon = \frac{D}{2R},$$

we get the critical curvature

$$R_c = \frac{E}{2\sigma_c}D$$

Good: R_c does not depend on L. Bad: experiments do not show linear dependence on D.

Question

Why we got wrong scaling?

Answer

We neglected stress concentration effects at the scale of flaws in the material

Griffith's criterion

Griffith's criterion

$\sigma_{\mathrm{c}}\sqrt{a} = \mathrm{const}$, where a is the typical size of flaws in the material.

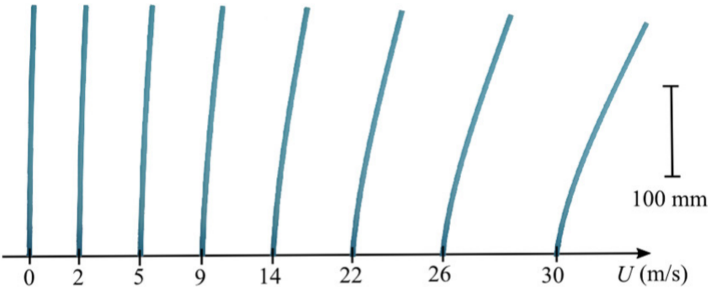
We assume $a \sim D$ and write

$$\sigma_{\mathrm{c}}\sqrt{D} = \frac{K_{\mathrm{Ic}}}{2\delta}$$

Then

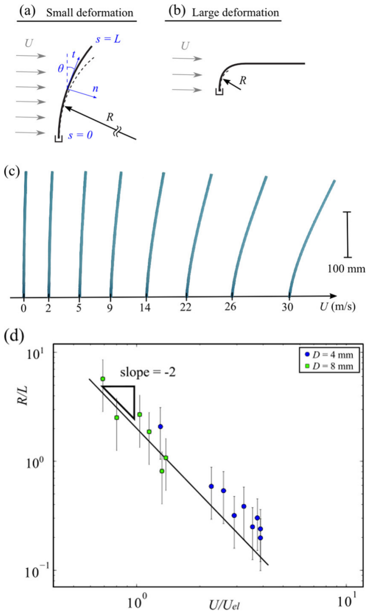
$$R_{\mathrm{c}} = \frac{\delta E}{K_{\mathrm{Ic}}} D^{\frac{3}{2}}$$

Experiment II



Experiments conducted in a wind tunnel on commercial straws

Critical wind speed

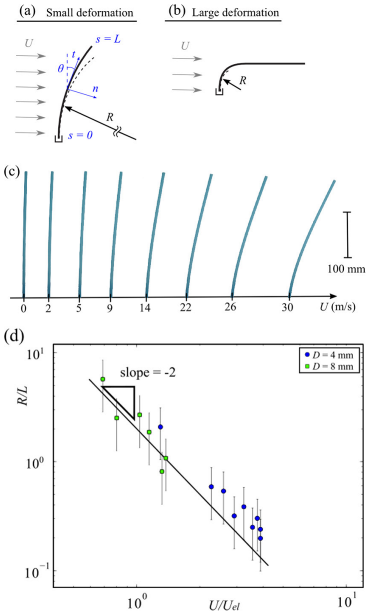


The wind force per unit length of the trunk

$$\mathbf{K} = \frac{1}{2}\rho_{\mathrm{air}} c_d D U (\mathbf{U} \cdot \mathbf{n})\mathbf{n}$$

where **n** is the unit vector normal to the trunk.

Critical wind speed



If the rod is weakly bent then the balance of the bending moment gives

$$\frac{EI}{R} \sim \frac{1}{2}\rho_{\mathrm{air}} C_d U^2 D L^2$$

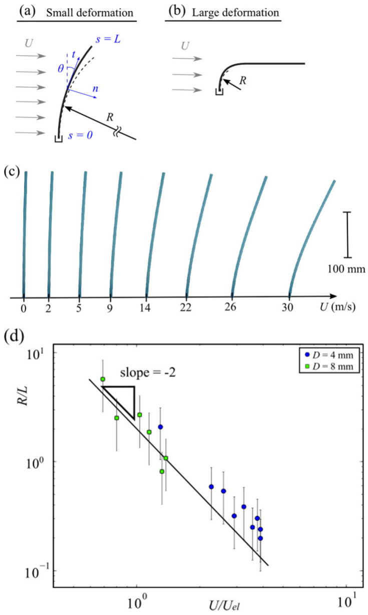
Then

$$\frac{R}{L} \sim \left(\frac{U_{\mathrm{el}}}{U}\right)^2$$

where

$$U_{\mathrm{el}} = \sqrt{\frac{2EI}{\rho_{\mathrm{air}} C_d D L^3}}$$

Critical wind speed



If the rod is strongly bent, the radius of curvature plays the role of a lever arm and the balance of moments yields

$$\frac{EI}{R} \sim \frac{1}{2}\rho_{\mathrm{air}} C_d U^2 D R^2$$

Thus

$$\frac{R}{L} \sim \left(\frac{U_{\mathrm{el}}}{U}\right)^{\frac{2}{3}}$$

Critical wind speed

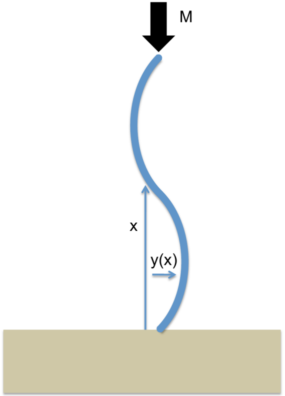
Using critical curvature

$$R_c = \frac{\delta E}{K_{\mathrm{Ic}}} D^{\frac{3}{2}}$$

we obtain the critical wind speed

$$U_c \sim \sqrt{\frac{K_{\mathrm{Ic}}}{\rho_{\mathrm{air}} \delta c_d}} \frac{D^{\frac{3}{4}}}{L}$$

Tree allometry



A tree limits its height at about 1/4 the critical buckling height under their own weight.

$$D \sim \sqrt{\frac{\rho_{\mathrm{s}} g}{E}} L^{\frac{3}{2}}$$

The ratio

$$\frac{\rho_{\mathrm{s}}}{E}$$

is approximately constant in trees. Thus

$$D \sim \beta L^{\frac{3}{2}}$$

Critical wind speed II

We can rewrite the critical wind speed as a function of the tree height only

$$U_{\mathrm{c}} \sim \sqrt{\frac{K_{\mathrm{Ic}}\beta^{\frac{3}{2}}}{\rho_{\mathrm{air}}\delta C_d}} L^{\frac{1}{8}}$$

Conclusion

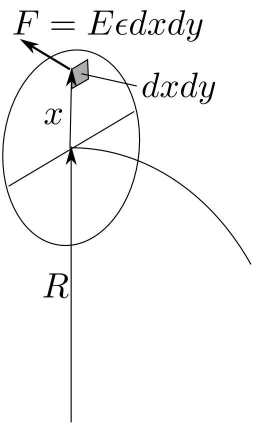
The critical wind speed has a very weak dependency on the tree size!

Summary

By combining Hooke's law, Griffith's criterion, and tree allometry, we deduced a critical wind speed which weakly depends on tree characteristics. This result is consistent with field measurements performed after storms. The absolute value of critical wind speed, found to be on the order of the maximal wind speeds expected on the Earth ($50\,\mathrm{m/s}$). Hence our results might contribute to understanding why trees are such old living systems.

Thank you for your attention!

Calculation of the bending moment



Strain

$$\epsilon = \frac{x}{R}$$

Stress

$$\sigma = E\epsilon$$

Force

$$\mathrm{d}F = \sigma \mathrm{d}x\mathrm{d}y$$

Moment

$$\mathrm{d}M = x\mathrm{d}F$$

The full beding moment

$$M = \int \mathrm{d}M = \int E\frac{x^2}{R}\mathrm{d}x\mathrm{d}y = \frac{EI}{R}$$