Journal Club: Experimental nonlocal and surreal Bohmian trajectories

Julius Ruseckas

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Institute of Theoretical Physics and Astronomy, Vilnius University

RESEARCH ARTICLE

QUANTUM MECHANICS

Experimental nonlocal and surreal Bohmian trajectories

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Dylan H. Mahler,[1,2]* Lee Rozema,[1,2] Kent Fisher,[3] Lydia Vermeyden,[3] Kevin J. Resch,[3] Howard M. Wiseman,[4]* Aephraim Steinberg[1,2]

Weak measurement allows one to empirically determine a set of average trajectories for an ensemble of quantum particles. However, when two particles are entangled, the trajectories of the first particle can depend nonlocally on the position of the second particle. Moreover, the theory describing these trajectories, called Bohmian mechanics, predicts trajectories that were at first deemed "surreal" when the second particle is used to probe the position of the first particle. We entangle two photons and determine a set of Bohmian trajectories for one of them using weak measurements and postselection. We show that the trajectories seem surreal only if one ignores their manifest nonlocality.

Science Advances **2**, e1501466 (2016).

Outline

1. Weak measurements
2. Bohmian mechanics
3. Experimental Bohmian trajectiories

Weak measurements

Measurements in quantum mechanics

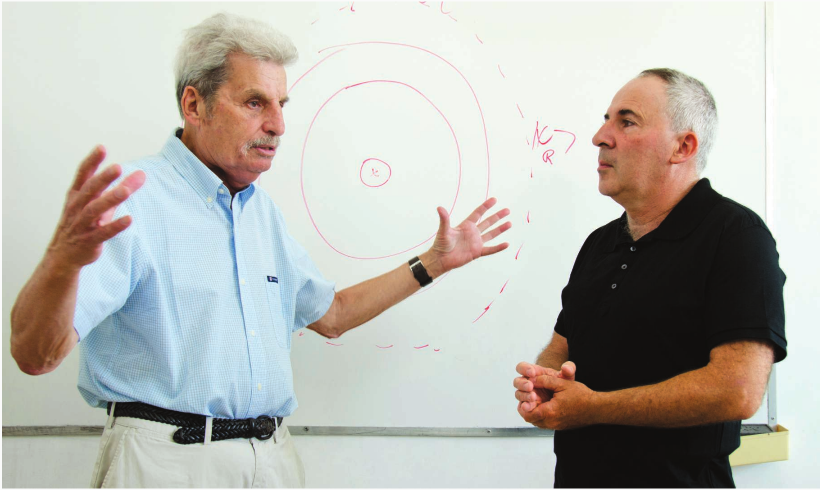
- A result is obtained after an interaction of the measuring apratus with a single quantum system
- Possible results of the measurement are given by the eigenvalues of the corresponding operator
- Strong interaction between the measuring aparatus and the quantum system
- Large change in the state of the quantum system—wave function collapse

What if the strength of the interaction is weak ?

It is possible to get an information about quantum system even when the interaction between the system and the aparatus is weak—weak measurement.

A large ensemble of identical quantum systems is needed.

Weak measurements



Yakir Aharonov (left) and Lev Vaidman

Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. **60**, 1351 (1988).

Hamiltonian describing the interaction between the quantum system and the measurement aparatus

$$\hat{H}_{\mathrm{I}} = \lambda \hat{q} \hat{A}$$

- $\hat{A}$ is the operator acting on the system
- $\hat{q}$ is the position of the measurement aparatus
- $\lambda \to 0$ is the interaction strength

The measurement aparatus initially is in the state $|\Phi(0)\rangle$ and the measured system in the state $|\psi\rangle$.

Weak measurements

- The measurements are performed on an ensemble of identical systems, each of which is prepared in the same initial state. The readings of the detectors are collected and averaged.
- Action of the Hamiltonian $\hat{H}_{\mathrm{I}}$ results in a small change of the mean momentum of the measuring aparatus $\langle \hat{p}_q \rangle_\tau - \langle \hat{p}_q \rangle_0$.
- We define the “weak value” of the operator $\hat{A}$ as

$$A^{\mathrm{w}} \equiv \frac{\langle \hat{p}_q \rangle_0 - \langle \hat{p}_q \rangle_\tau}{\lambda \tau}$$

- In the limit $\lambda \to 0$:

$$A^{\mathrm{w}} = \langle \psi | \hat{A} | \psi \rangle$$

We can consider the following procedure:

1. The momentum p_q of the measuring aparatus after the interaction is measured for each system in the ensemble.
2. The final, post-selection measurement on the ensemble is performed. The post-selection measurement determines wheter the quantum system is in the state $|b\rangle$.
3. The outcomes p_q only for the systems found in the state $|b\rangle$ are colleced and used to calculate the average $\langle \hat{p}_q \rangle_b$.

Similar to the previous definition,

$$A_b^{\mathrm{w}} = \frac{\langle \hat{p}_q \rangle_0 - \langle \hat{p}_q \rangle_b}{\lambda \tau}$$

is the weak value of of the operator $\hat{A}$ with the condition that the system is found in the state $|b\rangle$.

Using conditional probability

$$\langle \hat{p}_q \rangle_b = \frac{\langle \Psi(\tau) | \hat{p}_q \otimes |b\rangle\langle b| \Psi(\tau) \rangle}{\langle \Psi(\tau) | b \rangle \langle b | \Psi(\tau) \rangle}$$

Post-selection

Even in the limit $\lambda \to 0$ the average $\langle \hat{p}_q \rangle_b$ depends on the initial state of the detector and is a sum of two terms containing the anticommutator $\{\hat{A}, |b\rangle\langle b|\}$ and the commutator $[\hat{A}, |b\rangle\langle b|]$.

We can combine those two parts into one complex-valued quantity

$$A_b^{\mathrm{w}} = \frac{\langle b|\hat{A}|\psi\rangle}{\langle b|\psi\rangle}$$

Experimental realization

Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer

Sacha Kocsis,[1,2]* Boris Braverman,[1]* Sylvain Ravets,[3]* Martin J. Stevens,[4] Richard P. Mirin,[4] L. Krister Shalm,[1,5] Aephraim M. Steinberg[1]†

A consequence of the quantum mechanical uncertainty principle is that one may not discuss the path or "trajectory" that a quantum particle takes, because any measurement of position irrevocably disturbs the momentum, and vice versa. Using weak measurements, however, it is possible to operationally define a set of trajectories for an ensemble of quantum particles. We sent single photons emitted by a quantum dot through a double-slit interferometer and reconstructed these trajectories by performing a weak measurement of the photon momentum, postselected according to the result of a strong measurement of photon position in a series of planes. The results provide an observationally grounded description of the propagation of subensembles of quantum particles in a two-slit interferometer.

In classical physics, the dynamics of a particle's evolution are governed by its position and velocity; to simultaneously know the particle's position and velocity is to know its past, present, and future. However, the Heisenberg uncertainty principle in quantum mechanics forbids simultaneous knowledge of the precise position and velocity of a particle. This makes it impossible to determine the trajectory of a single quantum particle in the same way as one would that of a classical particle: Any information gained about the quantum particle's position irrevocably alters its momentum (and vice versa) in a way that is fundamentally uncertain. One consequence is that in Young's double-slit experiment one cannot determine through which slit a particle passes (position) and still observe interference effects on a distant detection screen (equivalent to measuring the momentum). Particle-like trajectories and wavelike interference are "complementary" aspects of the behavior of a quantum system, and an experiment designed to observe one neces-

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*These authors contributed equally to this work.
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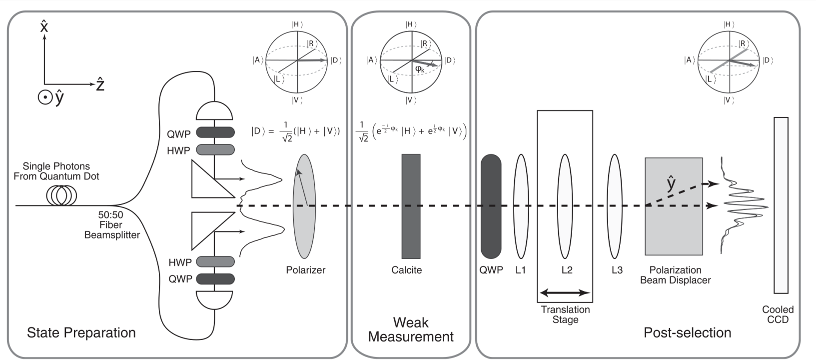
Science **332**, 1170 (2011).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|A⟩
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|V⟩
$|D\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$
$\frac{1}{\sqrt{2}}\left(e^{-\frac{i}{2}\varphi_k}|H\rangle + e^{\frac{i}{2}\varphi_k}|V\rangle\right)$
φ_k
QWP
HWP
Single Photons
From Quantum Dot
50:50
Fiber
Beamsplitter
HWP
QWP
Polarizer
Calcite
QWP
L1
L2
L3
Polarization
Beam Displacer
Translation
Stage
Cooled
CCD
State Preparation
Weak
Measurement
Post-selection

Experimental realization

- The polarization degree of freedom of the photons is used as a pointer that weakly couples to and measures the momentum of the photons.
- A polarizer prepares the photons with a diagonal polarization
$$|D\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$$
- The photons impinge upon the crystal of birefringent calcite with an incident angle θ that depends on their transverse momentum k_x.
- Crystal of calcite is $0.7\,\mathrm{mm}$-thick with its optic axis at $42°$ in the *x*-*z* plane.

Experimental realization

- $|H\rangle$ becomes the extraordinary polarization that encounters an angle-dependent index of refraction, $n_{\mathrm{e}}(\theta)$, and $|V\rangle$ becomes the ordinary polarization that encounters a constant index of refraction, n_{o}.
- Piece of calcite rotates the polarization state to

$$\frac{1}{\sqrt{2}}(\mathrm{e}^{-\mathrm{i}\varphi(k_x)/2}|H\rangle + \mathrm{e}^{\mathrm{i}\varphi(k_x)/2}|V\rangle)$$

- A quarter waveplate and a beam displacer are used to measure the polarization of the photons in the circular basis
- The momentum information encoded in polarization is transformed into an intensity difference between the two vertically displaced interference patterns.

Bohmian mechanics

In a classical mechanics the state of a system at time t is described by

- positions $\{q_1(t), q_2(t), \ldots, q_N(t)\}$
- velocities $\{v_1(t), v_2(t), \ldots, v_N(t)\}$

Bohmian mechanics describes the state of the quantum system by

- positions $\{q_1(t), q_2(t), \ldots, q_N(t)\}$
- wave function $\Psi(q_1, q_2, \ldots, q_N, t)$

Spin of a particle is included in the wave function Ψ.

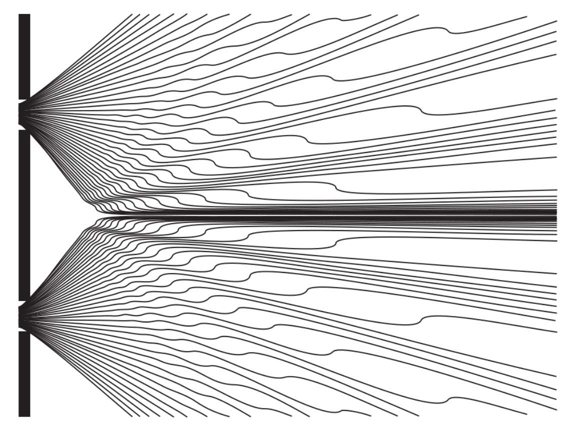
Equations of Bohmian mechanics

$$\frac{\mathrm{d}}{\mathrm{d}t}q_i = \frac{\hbar}{m_i}\operatorname{Im}\frac{\Psi^*\nabla_i\Psi}{\Psi^*\Psi}$$

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi = -\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\nabla_i^2\Psi + V\Psi$$

Quantum Equilibrium Hypothesis

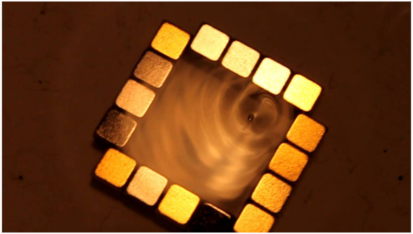
Whenever a system has wave function Ψ then its configuration is (or can be taken to be) random with probability distribution $|\Psi|^2$.



Bohmian trajectories in the double-slit experiment

Pilot-Wave Hydrodynamics

https://youtu.be/nmC0ygr08tE



https://youtu.be/72DA4fgamPE

Weak measurement of velocity:

$$v(x) = \lim_{\tau \to 0} \frac{1}{\tau} \left(x - \operatorname{Re} \frac{\langle x | \hat{U}(\tau) \hat{X} | \Psi \rangle}{\langle x | \hat{U}(\tau) | \Psi \rangle} \right)$$

where $\hat{U}(\tau) = \exp\left(-\frac{\mathrm{i}}{\hbar} \hat{H} \tau\right)$

H. M. Wiseman, New J. Phys. **9**, 165–177 (2007).

Experimental Bohmian trajectiories

Experimental trajectories of single photons

Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer

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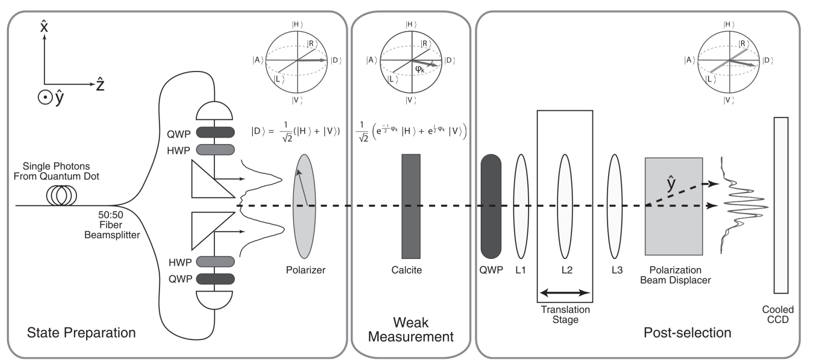
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|D⟩
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Single Photons From Quantum Dot
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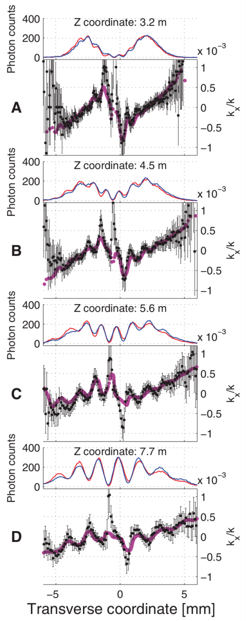
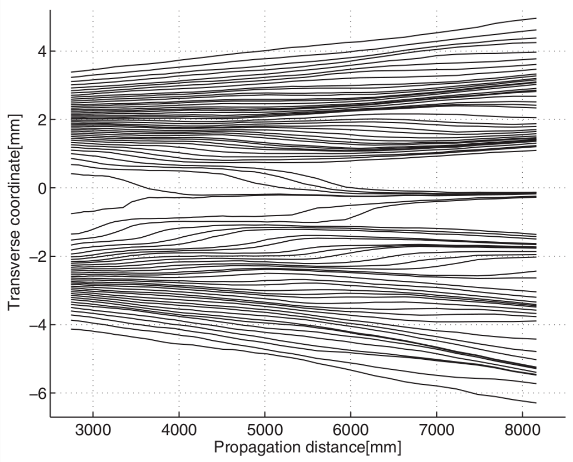
$\frac{1}{\sqrt{2}}\left(e^{\frac{-i}{2}\varphi_k}|H\rangle + e^{\frac{i}{2}\varphi_k}|V\rangle\right)$
φ_k
Polarizer
Calcite
QWP
L1
L2
L3
Polarization Beam Displacer
Translation Stage
Cooled CCD
State Preparation
Weak Measurement
Post-selection

- A system of three cylindrical lenses, with the middle lens translatable in the z direction, allows the initial slit function to be imaged over an arbitrary distance.
- The momentum information encoded in polarization is transformed into an intensity difference between the two vertically displaced patterns.
- The pixel on the CCD where each photon is detected corresponds to the photon's x position.

Reconstructed trajectories





Trajectories of entangled photons

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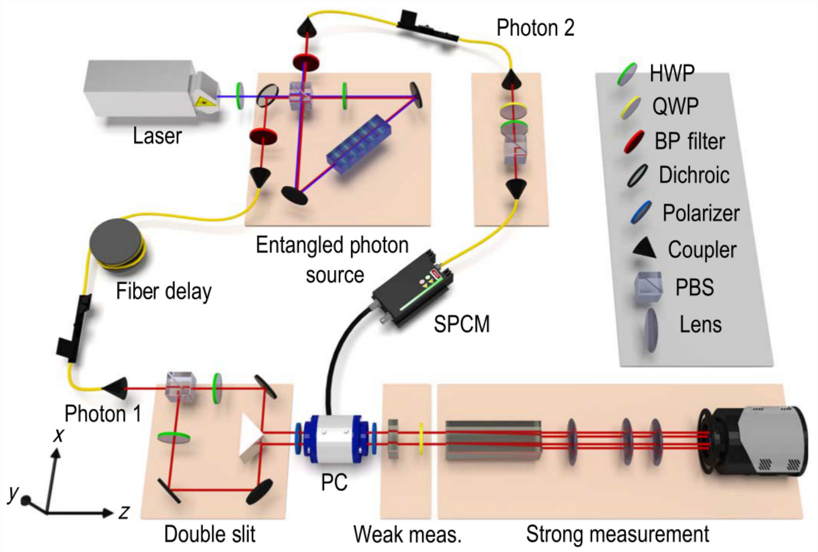
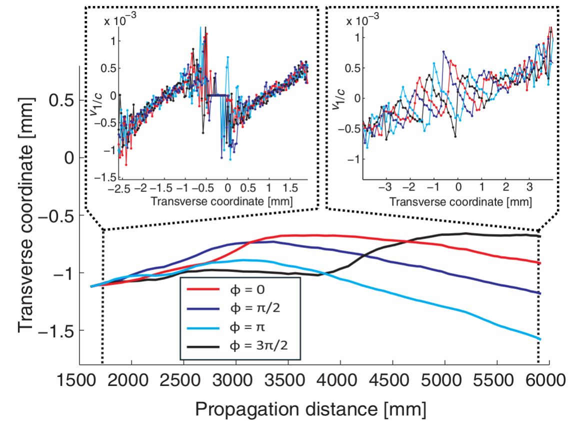

Photon 2
Laser
HWP
QWP
BP filter
Dichroic
Polarizer
Coupler
PBS
Lens
Entangled photon source
Fiber delay
SPCM
Photon 1
x
y
z
PC
Double slit
Weak meas.
Strong measurement

Experimental setup

- Two photons are prepared in a maximally entangled state
- Due to the polarizing beamsplitter, photon 1 has horizontal polarization in the upper slit and vertical polarization in the lower slit.
- Because of the initial polarization entanglement, the path of photon 1 is now entangled with the polarization of photon 2
- A Pockels cell blocks photon 1 unless photon 2 was detected
- The trajectories of a single photon 1 are measured, postselected on a detection of photon 2

The trajectory of photon 1 is affected by the remote choice of how to measure photon 2



The joint state of the double-slit particle and another quantum system with a spin degree of freedom

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\int \mathrm{d}x_1\mathrm{d}x_2|x_1\rangle|x_2\rangle$$
$$\times\left(\Psi_\mathrm{u}(x_1,t)\phi_\mathrm{H}(x_2,t)|\mathrm{H}\rangle + \Psi_\mathrm{l}(x_1,t)\phi_\mathrm{V}(x_2,t)|\mathrm{V}\rangle\right)$$

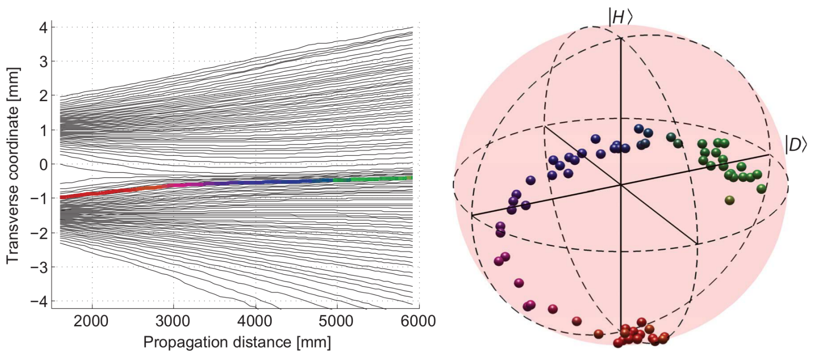
Classical intuition

Measurement of the spin of particle 2 should indicate through which slit particle 1 had gone

If, at the time of the measurement of the spin of particle 2, particle 1 is in

- the *near field* of the double-slit apparatus $\rightarrow$ the measurement outcome is perfectly correlated with the origin of each Bohmian trajectory
- the *far field* $\rightarrow$ the trajectories predicted by Bohmian mechanics often fail to agree with the outcome of the measurement

The trajectories of photon 1 are measured without performing a postselection on photon 2



Polarization of photon 2 along a trajectory of photon 1

Indeed, our observation of the change in polarization of a free space photon, as a function of the time of measurement of a distant photon (along one reconstructed trajectory), is an exceptionally compelling visualization of the nonlocality inherent in any realistic interpretation of quantum mechanics.

Thank you for your attention!