Light-induced Abelian and non-Abelian gauge potentials for ultracold atoms

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Outline

1. Motivation
2. Some aspects of adiabatic approximation
3. Abelian effective potentials for Λ-type atoms
   - using light beams with orbital angular momentum
   - using counterpropagating light beams
4. Non-Abelian effective potentials for tripod coupling scheme

Why effective magnetic field for atoms?

Atomic physics $\Longleftrightarrow$ Solid state physics:

- Degenerate Fermi gas $\Longleftrightarrow$ Electrons in solids
- Atoms in optical lattices

Advantages and disadvantages of cold atoms

- **Advantage:** Freedom in changing experimental parameters that are often inaccessible in standard solid state physics
- **Disadvantage:** Trapped atoms are electrically neutral particles. Direct analogy with magnetic properties of solids is not necessarily straightforward

Ways to create effective magnetic field for cold atoms

- Rotation — usual method to create effective magnetic field
  - Constant effective magnetic field $B_{\mathrm{eff}} \sim \Omega$
  - Trapping frequency $\omega_{\mathrm{eff}} = \omega - \Omega$
  - Effective magnetic field acts on atoms in the same way
- Optical lattices having assimetry in the atomic transitions between the lattice sites.
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Our work: another method of creating effective potentials

Effective gauge potentials can be created using light beams with the orbital angular momentum in the EIT configuration.

Advantages

- No rotation
- No need for optical lattice

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- J. Ruseckas, G. Juzeliūnas, P. Öhberg, and M. Fleischhauer Phys. Rev. Lett. **95**, 010404 (2005)
- P. Öhberg, G. Juzeliūnas, J. Ruseckas, and M. Fleischhauer, preprint arXiv: cond-mat/0509766; accepted for publication in Phys. Rev. A
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Adiabatic Approximation

- The full atomic Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2M} + \hat{V}(\mathbf{r}) + \hat{H}_0(\mathbf{r}, t).$$

  - $\hat{H}_0(\mathbf{r}, t)$ — the Hamiltonian for the electronic (fast) degrees of freedom,
  - $\hat{p}^2/2M + \hat{V}(\mathbf{r})$ — the Hamiltonian for center of mass (slow) degrees of freedom.
  - $\hat{V}(\mathbf{r})$ — the external trapping potential.
- $\hat{H}_0(\mathbf{r}, t)$ has eigenfunctions $|\chi_n(\mathbf{r}, t)\rangle$ with eigenvalues $\varepsilon(\mathbf{r}, t)$.
- Full atomic wave function

$$|\Phi\rangle = \sum_n \Psi_n(\mathbf{r}, t)|\chi_n(\mathbf{r}, t)\rangle.$$

Adiabatic Approximation

Substituting into the Schrödinger equation $i\hbar\partial/\partial t|\Phi\rangle = \hat{H}|\Phi\rangle$ one can write the equation for the coefficients $\Psi_n(\mathbf{r}, t)$ in the form

$$i\hbar\frac{\partial}{\partial t}\Psi = \left[\frac{1}{2M}(-i\hbar\nabla - \mathbf{A})^2 + V + \dot{\beta}\right]\Psi,$$

where

$$\Psi = \begin{pmatrix} \Psi_1 \\ \cdots \\ \Psi_n \end{pmatrix},$$

$$\mathbf{A}_{n,n'} = i\hbar\langle\chi_n(\mathbf{r}, t)|\nabla\chi_{n'}(\mathbf{r}, t)\rangle,$$

$$V_{n,n'} = \varepsilon(\mathbf{r}, t)\delta_{n,n'} + \langle\chi_n(\mathbf{r}, t)|\hat{V}(\mathbf{r})|\chi_{n'}(\mathbf{r}, t)\rangle,$$

$$\beta_{n,n'} = -i\hbar\int dt\langle\chi_n(\mathbf{r}, t)|\frac{\partial}{\partial t}\chi_{n'}(\mathbf{r}, t)\rangle.$$

Adiabatic Approximation

Non-degenerate states

The first state is well separated from the rest. Off-diagonal terms are neglected.

$$i\hbar\frac{\partial}{\partial t}\Psi_1 = \left[\frac{1}{2M}(-i\hbar\nabla - \mathbf{A})^2 + V + \phi\right]\Psi_1,$$

where

$$\mathbf{A} = \mathbf{A}_{1,1},$$
$$V = V_{1,1},$$
$$\phi = \frac{1}{2M}\sum_{n\neq 1}\mathbf{A}_{1,n}\cdot\mathbf{A}_{n,1} + \dot{\beta}_{1,1}.$$

Adiabatic Approximation

Degenerate states

The first q dressed states are degenerate and these levels are well separated from the remaining $N - q$

$$i\hbar \frac{\partial}{\partial t} \tilde{\Psi} = \left[\frac{1}{2M} (-i\hbar \nabla - \mathbf{A})^2 + V + \phi \right] \tilde{\Psi},$$

where $\mathbf{A}$ and V are truncated $q \times q$ matrices,

$$\phi_{n,n'} = \frac{1}{2M} \sum_{m=q+1}^{N} \mathbf{A}_{n,m} \cdot \mathbf{A}_{m,n'} + \dot{\beta}_{n,n'}.$$

The effective vector potential $\mathbf{A}$ is called the Berry connection.

Gauge Transformations

Non-degenerate states

We have freedom of choosing the phase of the adiabatic states

$$|\chi_n(\mathbf{r}, t)\rangle \to e^{-\frac{i}{\hbar} u_n(\mathbf{r},t)} |\chi_n(\mathbf{r}, t)\rangle .$$

The transformation of the potentials

$$\mathbf{A} \to \mathbf{A} + \nabla u_1 ,$$
$$\phi \to \phi - \frac{\partial}{\partial t} u_1 .$$

Gauge Transformations

Degenerate states

The adiabatic basis can be changed by a local unitary transformation $U(\mathbf{r}, t)$

$$\tilde{\Psi} \to U(\mathbf{r}, t)\tilde{\Psi}.$$

The transformation of the potentials

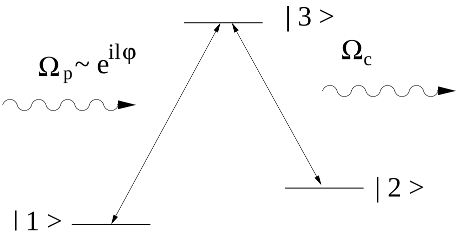
$$\mathbf{A} \to U\mathbf{A}U^\dagger - i\hbar(\nabla U)U^\dagger,$$

$$\phi \to U\phi U^\dagger + i\hbar\frac{\partial U}{\partial t}U^\dagger.$$

The Berry connection **A** is related to a curvature **B** as

$$B_i = \frac{1}{2}\epsilon_{ikl}F_{kl}, \qquad F_{kl} = \partial_k A_l - \partial_l A_k - \frac{i}{\hbar}[A_k, A_l].$$

Λ-type Atoms



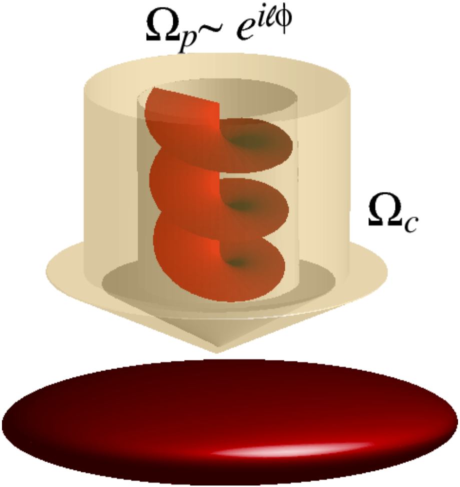
Probe beam: $\Omega_p = \mu_{13} E_p$
Control beam: $\Omega_c = \mu_{23} E_c$

Dark state

$$|D\rangle \sim \Omega_c |1\rangle - \Omega_p |2\rangle$$

Destructive interference, cancelation of absorbtion — EIT

Light Vortices



Light vortex

Light vortex — light beam with phase

$$e^{ikz+il\varphi},$$

where φ is azimuthal angle, l — winding number.

Light vortices have orbital angular momentum (OAM) along the propagation axis $M_z = \hbar l$.

Effective Magnetic Field

$$\mathbf{A} = -\hbar \frac{|\zeta|^2}{1+|\zeta|^2} \nabla S, \qquad \mathbf{B} = \hbar \frac{\nabla S \times \nabla |\zeta|^2}{(1+|\zeta|^2)^2},$$

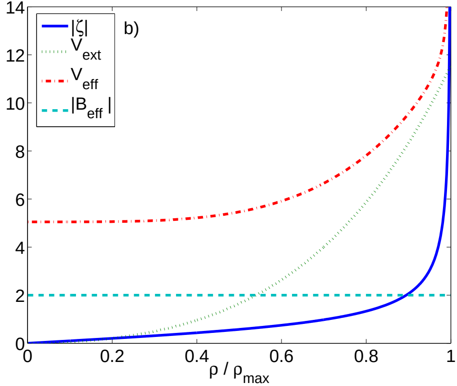
$$\phi = \frac{\hbar^2}{2M} \frac{(\nabla |\zeta|)^2 + |\zeta|^2 (\nabla S)^2}{(1+|\zeta|^2)^2},$$

where

$$\zeta = \frac{\Omega_p}{\Omega_c} = |\zeta| e^{iS}, \qquad S = l\varphi.$$

- Light beams with OAM can introduce an effective magnetic field which acts on the electrically neutral atoms.
- The vector potential **A** is determined by:
  - the gradient of phase of the probe beam,
  - the ratio between the intensities of the control and probe beams.

Constant Effective Magnetic Field



The effective trapping potential $V_{\mathrm{eff}} = V + \phi$ corresponding to the case where the effective magnetic field **B** is constant.

Magnetic Monopole?

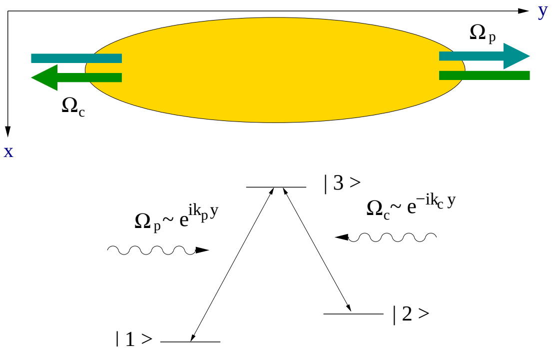
$$\mathbf{A} = -\frac{\hbar l}{2}\frac{1-\cos\theta}{r\sin\theta}\mathbf{e}_{\varphi}, \qquad \mathbf{B} = -\frac{\hbar l}{2r^2}\mathbf{e}_r, \qquad \phi = \frac{\hbar^2}{2M}\frac{l^2+1}{4r^2}.$$

- The Rabi frequencies should obey the following:

$$|\Omega_p|^2 = f(\mathbf{r})(1-\cos\theta), \qquad |\Omega_c|^2 = f(\mathbf{r})(1+\cos\theta).$$

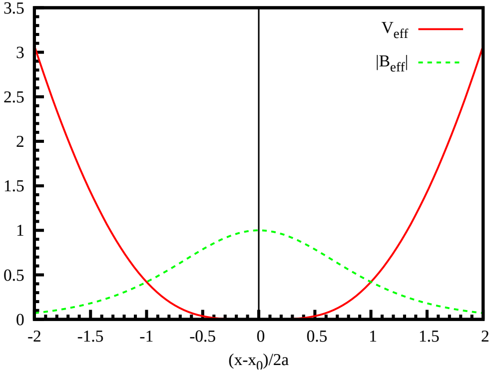
- The effective field necessary differs from the field of a monopole in the vicinity of the negative (or positive) part of the z-axis.
- Field of a magnetic monopole cannot be created in the whole space.

Counterpropagating Light Beams



The phase $S = (k_p + k_c)y$

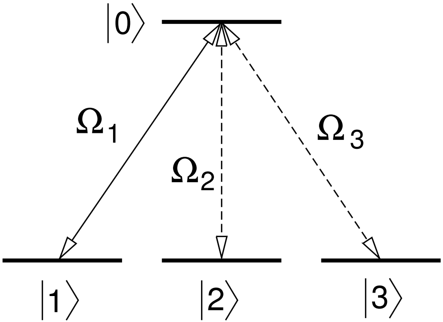
Counterpropagating Gaussian Beams



Effective trapping potential V_{eff} and effective magnetic field B_{eff} produced by counter-propagating Gaussian beams.

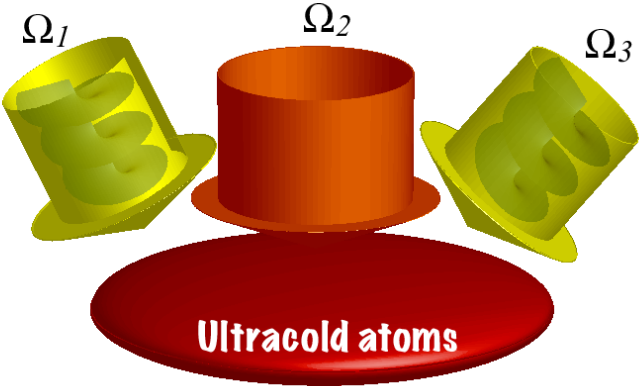
Characteristic width $a = \sigma^2/4\Delta$

Tripod Coupling Scheme



- Two degenerate dark states
- Non-Abelian gauge potentials

Tripod Coupling Scheme



Tripod Coupling Scheme

- Two degenerate dark states:

$$|D_1\rangle = \sin\phi e^{iS_{31}}|1\rangle - \cos\phi e^{iS_{32}}|2\rangle,$$
$$|D_2\rangle = \cos\theta\cos\phi e^{iS_{31}}|1\rangle + \cos\theta\sin\phi e^{iS_{32}}|2\rangle - \sin\theta|3\rangle,$$

where

$$\Omega_1 = \Omega\,\sin\theta\,\cos\phi\,e^{iS_1},\quad \Omega_2 = \Omega\,\sin\theta\,\sin\phi\,e^{iS_2},\quad \Omega_3 = \Omega\,\cos\theta\,e^{iS_3}.$$

- Vector gauge potential:

$$\mathbf{A}_{11} = \hbar\left(\cos^2\phi\nabla S_{23} + \sin^2\phi\nabla S_{13}\right),$$
$$\mathbf{A}_{12} = \hbar\cos\theta\left(\frac{1}{2}\sin(2\phi)\nabla S_{12} - i\nabla\phi\right),$$
$$\mathbf{A}_{22} = \hbar\cos^2\theta\left(\cos^2\phi\nabla S_{13} + \sin^2\phi\nabla S_{23}\right).$$

Magnetic Monopole

- Laser fields:

$$\Omega_{1,2} = \Omega_0 \frac{\rho}{R} e^{i(kz \mp \varphi)}, \qquad \Omega_3 = \Omega_0 \frac{z}{R} e^{ik' x}.$$

- The effective magnetic field

$$\mathbf{B} = \frac{\hbar}{r^2} \mathbf{e}_r \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \cdots .$$

Summary

- Light beams with relative orbital angular momentum can introduce an effective magnetic field acting on the electrically neutral atoms.
- Effective magnetic field can be shaped by choosing proper control and probe beams.
- The method can be extended to non-Abelian gauge potentials.
- Artificial magnetic phenomena in ultra-cold atomic gases.

Thank you!