Multicomponent slow and stationary polaritons in atomic gases

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Abstract

During the last several years there has been a great deal of interest in slow and stationary light which can be described using the concept of dark-state polaritons (DSPs) [1,2]. The slow and stationary DSPs are quasiparticles composed of light and atomic spin excitations. They are formed in resonant atomic media under the influence of several light beams of higher intensities [1-3]. In the previous studies it was shown that the stationary DSPs can be produced by applying two counter-propagating control beams in a double Lambda setup [1,2]. Such polaritons behave as bosons with a finite effective mass and can even experience the Bose-Einstein condensation [2]. Yet the existing studies restrict to polaritons which are described by a single-component field, there being no previous treatment of multi-component DSPs.

Here we propose a more complex setup involving four control laser beams and two probe beams. This makes it possible to create the two component slow and stationary polaritons exhibiting a number of distinct properties, such as neutrino type oscillations between the probe fields. Under certain conditions the DSPs can be described by a Dirac-type equation for a relativistic particle with a finite mass. This leads to the "particle-antiparticle"' dispersion branches separated by a gap δ. The corresponding Compton wave-length $\lambda_C = v_0/\delta$ determines the tunneling length of probe light though the sample, v_0 being the "ultrarelativisitc" velocity of slow polaritons.

[1] F. E. Zimmer, J. Otterbach, R. G. Unanyan, B. W. Shore, and M. Fleischhauer, Phys. Rev. A **77**, 063823 (2008).

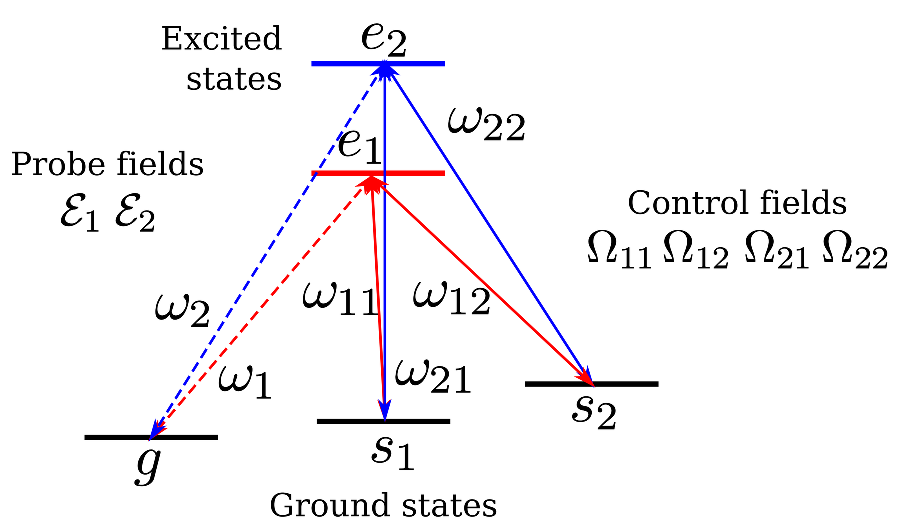
[2] M. Fleischhauer, J. Otterbach, and R. G. Unanyan, Phys. Rev. Lett. **101**, 163601 (2008).

[3] J. Ruseckas, G. Juzelinas, P. Öhberg, and S. M. Barnett, Phys. Rev. A **76**, 053822 (2007).

Formulation

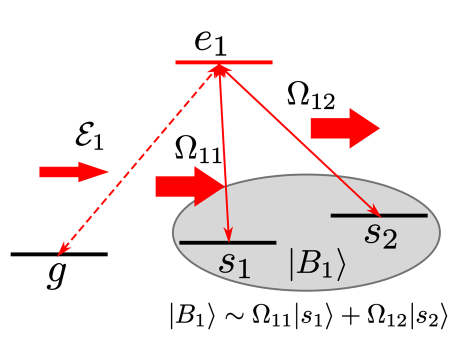
Conditions:

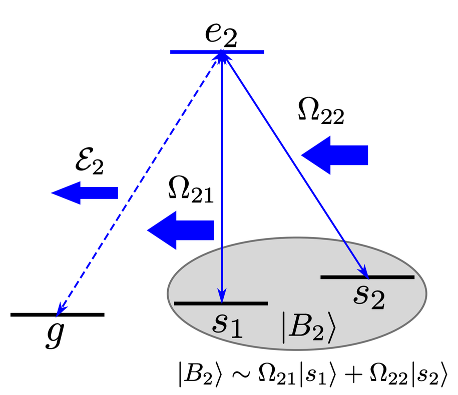
- Propagation of two weak (probe) light beams in a resonant atomic medium under the influence of several stronger (control) laser beams.
- Probe beams couple the ground state g to two excited states e_1 and e_2 .
- Control beams couple excited states e_j to another two ground states s_k .
- Control beams make the medium transparent for resonant probe beams in a narrow frequency range due to the electromagnetically induced transparency (EIT).
- 4-photon resonance: $\omega_1 - \omega_{11} = \omega_2 - \omega_{21}$, $\omega_1 - \omega_{12} = \omega_2 - \omega_{22}$.



Limiting cases:

- $\langle B_1|B_2\rangle = 0$ — two not connected tripods
- $\langle B_1|B_2\rangle = 1$ — double Lambda setup
- $0 < |\langle B_1|B_2\rangle| < 1$ — two connected tripods





Matrix representation: $\mathcal{E} = \begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_1 \end{pmatrix}$, $\Phi_s = \begin{pmatrix} \Phi_{s1} \\ \Phi_{s2} \end{pmatrix}$, $\Phi_e = \begin{pmatrix} \Phi_{e1} \\ \Phi_{e2} \end{pmatrix}$

Equation for the slowly (in time) varying amplitudes of the probe fields:

$$\partial_t \mathcal{E} - \frac{i}{2} c\hat{k}^{-1}\nabla^2 \mathcal{E} - \frac{i}{2} c\hat{k}\mathcal{E} = ig\Phi_g^* \Phi_e$$

Equations for the atomic probability amplitudes (neglecting atomic motion):

$$\begin{aligned} i\hbar\partial_t \Phi_g &= -\hbar g \mathcal{E}^\dagger \Phi_e \\ i\hbar\partial_t \Phi_e &= \hbar\hat{\Delta}\Phi_e - \hbar\hat{\Omega}\Phi_s - \hbar g \Phi_g \mathcal{E} \\ i\hbar\partial_t \Phi_s &= \hbar\hat{\delta}\Phi_s - \hbar\hat{\Omega}^\dagger \Phi_e \end{aligned}$$

Here the matrices are $\hat{k} = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$, $\hat{\Omega} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}$, $\hat{\Delta} = \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{pmatrix}$, $\hat{\delta} = \begin{pmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{pmatrix}$

$g = g_j = \mu_j(\omega_j/2\epsilon_0\hbar)^{1/2}$ is the coupling strength; Φ_g and Φ_e are the atomic probability amplitudes of the ground and excited states; $k_1 = \omega_1/c$ and $k_2 = \omega_2/c$ are wave-numbers; Δ_1 and Δ_2 are the detunings from one-photon resonance; δ_1 and δ_2 are the detunings from two-photon resonance.

The adiabatic approximation (neglection of the excited state population): $\Phi_s = -g\Phi_g\hat{\Omega}^{-1}\mathcal{E}$

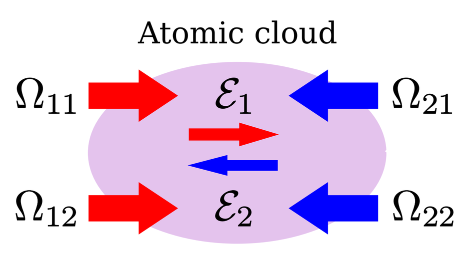
Matrix equation for the two-component probe field (for stationary control fields):

$$\boxed{v^{-1}\frac{\partial}{\partial t}\mathcal{E} - \frac{i}{2}\hat{k}^{-1}\nabla^2\mathcal{E} - \frac{i}{2}\hat{k}\mathcal{E} + i\left(v^{-1} - \frac{1}{c}\right)\hat{D}\mathcal{E} = 0}$$

$\hat{D} = \hat{\Omega}\hat{\delta}\hat{\Omega}^{-1}$ — matrix of the two-photon detuning

$v^{-1} = \frac{1}{c} + \frac{g^2 n}{c}(\hat{\Omega}^\dagger)^{-1}\hat{\Omega}^{-1}$ — defines matrix of inverse group velocity

Counter-propagating beams



Counter-propagating probe and control beams:
$\mathcal{E}_1(\mathbf{r},t) = \tilde{\mathcal{E}}_1(\mathbf{r},t)e^{ik_1 z}$, $\mathcal{E}_2(\mathbf{r},t) = \tilde{\mathcal{E}}_2(\mathbf{r},t)e^{-ik_2 z}$ and
$\Omega_{1j} = \tilde{\Omega}_{1j}e^{ik_{1j}z}$, $\Omega_{2j} = \tilde{\Omega}_{2j}e^{-ik_{2j}z}$.

Equation for paraxial probe beams neglecting the diffraction term:

$$\boxed{\tilde{v}^{-1}\frac{\partial}{\partial t}\tilde{\mathcal{E}} + \sigma_z\frac{\partial}{\partial z}\tilde{\mathcal{E}} + i\left(\tilde{v}^{-1} - \frac{1}{c}\right)\tilde{D}\tilde{\mathcal{E}} = 0}$$

The Rabi frequencies of control beams with equal intensities: $\tilde{\Omega}_{ij} = \frac{\Omega}{\sqrt{2}}\exp(iS_{ij})$

The relative phase of the control fields: $S = \frac{1}{2}(S_{12} + S_{21} - S_{11} - S_{22})$

- $S = \pm\pi/2$ — two not connected tripods
- $S = 0$ — double Lambda setup
- $0 < |S| < \pi/2$ — two connected tripods

Matrix of inverse polariton velocity: $\sigma_z\tilde{v}^{-1} \approx \frac{1}{v_0}\frac{1}{\sin^2 S}\begin{pmatrix} 1 & -\cos S \\ \cos S & -1 \end{pmatrix}$

Eigenvalues of $\sigma_z\tilde{v}^{-1}$: $\frac{1}{v_{\rm gr}^{\pm}} = \pm\frac{1}{v_0|\sin S|}$, $v_0 = \frac{c\Omega^2}{g^2 n}$

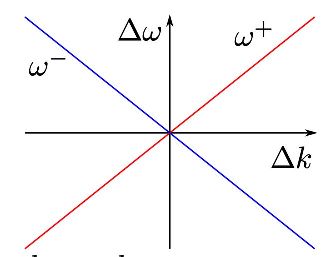
Eigenfunctions of $\sigma_z\tilde{v}^{-1}$: $\chi^{\pm} = \frac{1}{\sqrt{2(1\mp|\sin S|)}}\begin{pmatrix} \cos S \\ (1 \mp |\sin S|) \end{pmatrix}$

- The group velocity is a non-diagonal matrix (for $S \neq \pi/2$)
- Individual probe fields do not have a definite group velocity
- Only special combinations of both probe fields (polaritons) propagate in the atomic cloud with the definite velocities
- Neutrino-type oscillations are possible (next)

Neutrino type oscillations for polaritons

Zero two-photon detuning $\delta_1 = \delta_2 = 0$

$$\boxed{\sigma_z\tilde{v}^{-1}\frac{\partial}{\partial t}\tilde{\mathcal{E}} + \frac{\partial}{\partial z}\tilde{\mathcal{E}} = 0}$$

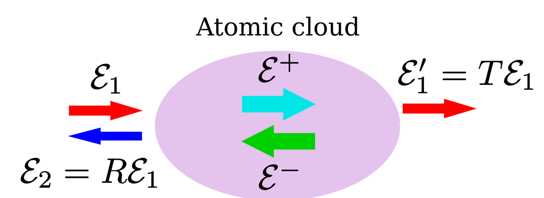


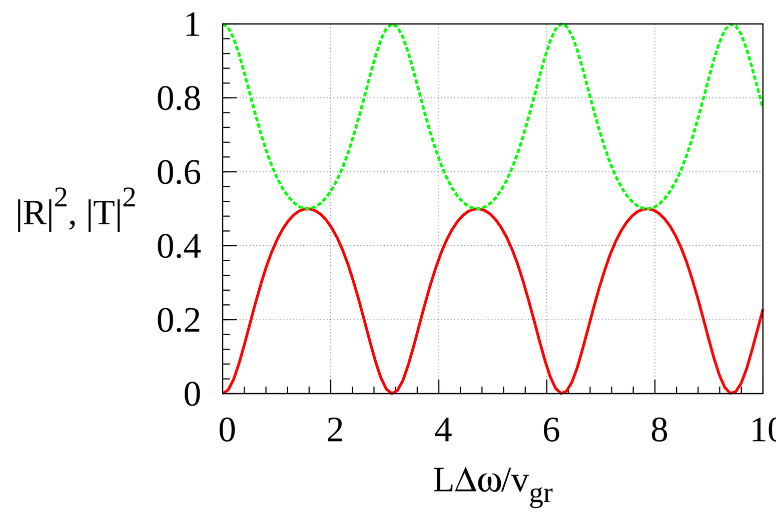
Two polaritons $\tilde{\mathcal{E}}^{\pm} \sim \chi^{\pm}$. Two dispersion branches with opposite slopes: $\Delta\omega^{\pm} = \pm v_{\rm gr}\Delta k$. Here $\Delta\omega = 0$ is the intersection point of both polariton branches.

$\mathcal{E}_1$ is reflected into $\mathcal{E}_2$. Reflection and transmission coefficients:

$$R = \frac{-2i\cos(S)\sin(\Delta\omega L/v_{\rm gr})}{(1-|\sin S|)e^{-i\Delta\omega\frac{L}{v_{\rm gr}}} - (1+|\sin S|)e^{i\Delta\omega\frac{L}{v_{\rm gr}}}}, \quad T = \frac{2|\sin S|}{(1+|\sin S|)e^{i\Delta\omega\frac{L}{v_{\rm gr}}} - (1-|\sin S|)e^{-i\Delta\omega\frac{L}{v_{\rm gr}}}}$$

Oscillations of R and T occur if $|S| \neq \pi/2$ i.e. if we have two connected tripod systems

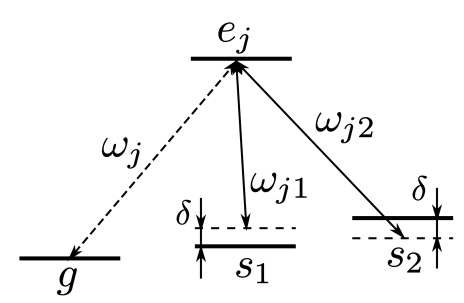




Dirac equation for two-component polariton

We assume that $S = \pm\pi/2$. Non-zero two photon detuning $\delta_1 = -\delta_2 \equiv \delta \neq 0$

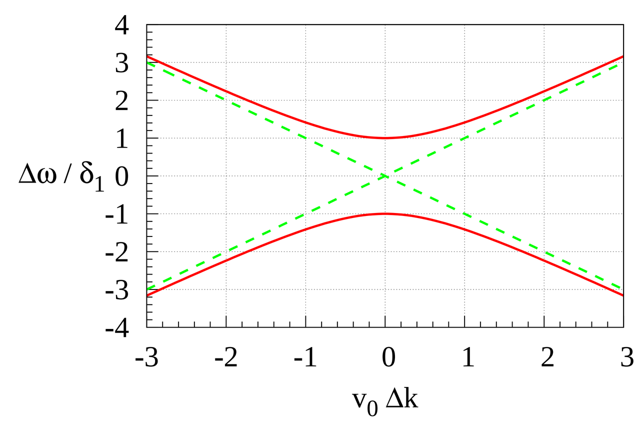
- A gap in dispersion ("electron-positron" type spectrum)
- Dirac type equation with non-zero mass for two component polaritons:

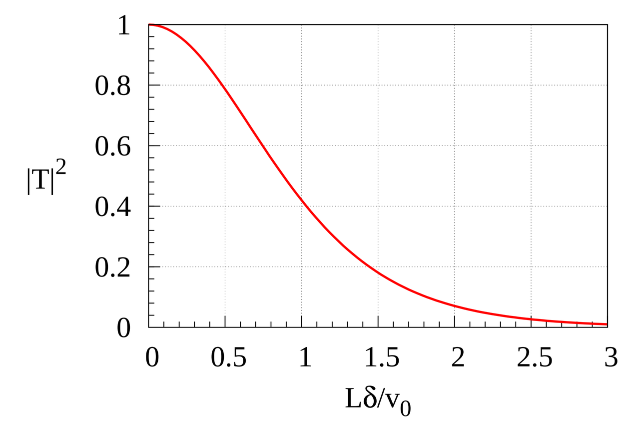


$$\boxed{i\frac{\partial}{\partial t}\tilde{\mathcal{E}} = -iv_0\sigma_z\frac{\partial}{\partial z}\tilde{\mathcal{E}} + \delta\sigma_y\tilde{\mathcal{E}}}$$

Relativistic particle-antiparticle dispersion: $\Delta\omega^{\pm} = \pm\sqrt{v_0^2\Delta k^2 + \delta^2}$

$\hbar\delta = mv_0^2$ — gap width, m — polariton effective mass





Reflection and transmission coefficients at the gap center ($\Delta\omega = 0$):

$$T = \cosh^{-1}(L/\lambda_C), \quad R = \tanh(L/\lambda_C)$$

$\lambda_C = \hbar/mv_0 = v_0/\delta$ — Compton wave-length of the polariton. The Compton wave-length determines the polariton tunneling length.