Adiabatic pulse propagation in multi-level systems

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Outline

Introduction


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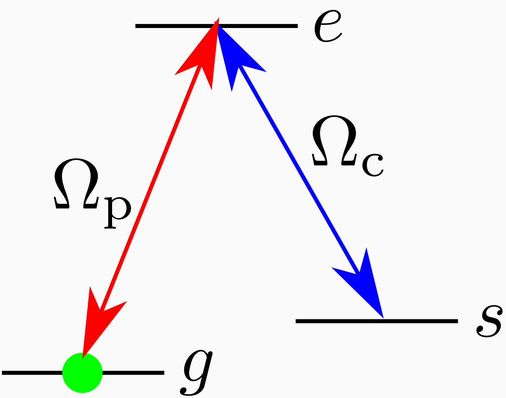
Cycling at the speed of light?



Probe beam: $\Omega_{\mathrm{p}} = \mu_{ge} E_{\mathrm{p}}$

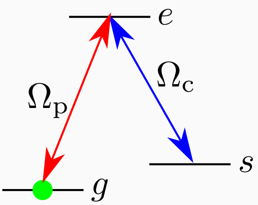
Control beam: $\Omega_{\mathrm{c}} = \mu_{ge} E_{\mathrm{c}}$

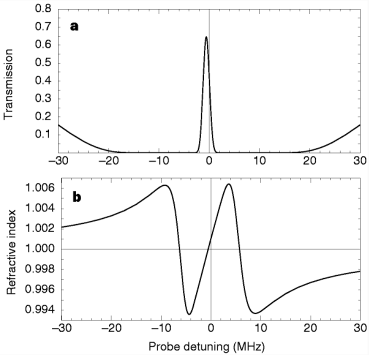
Three level Λ system

- Dark state

$$|D\rangle \sim \Omega_{\mathrm{c}}|g\rangle - \Omega_{\mathrm{p}}|s\rangle$$

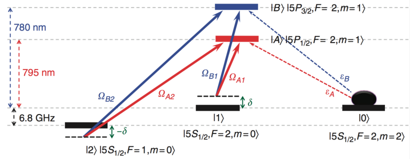
- Transitions $g \to e$ and $s \to e$ interfere destructively
- Cancelation of absorbtion
- Electromagnetically induced transparency—EIT
- Very fragile
- Very narrow transparency window

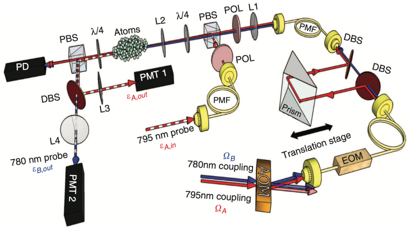




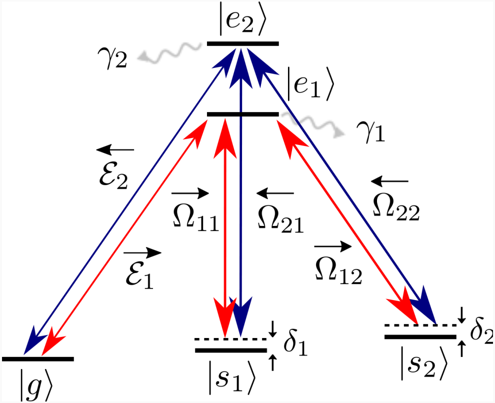
- Narrow transparency window $\Delta\omega \sim 1\,\mathrm{MHz}$
- Very dispersive medium
- Small group velocity — slow light

M.-J. Lee, J. Ruseckas, *et al*, Nat. Commun. **5**, 5542 (2014).

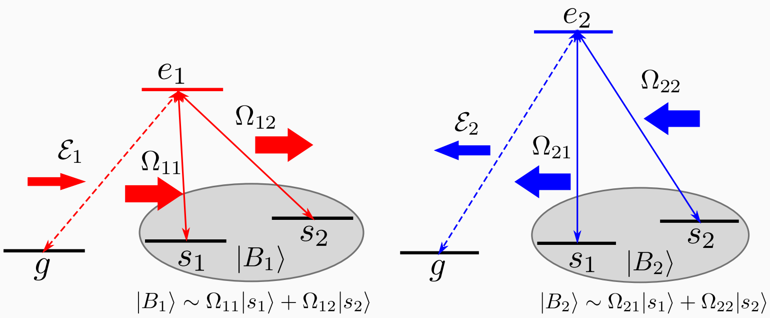




Double tripod setup



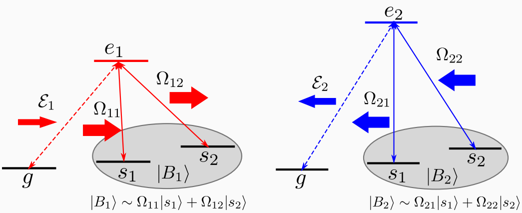
- R. G. Unanyan, J. Otterbach, M. Fleischhauer, J. Ruseckas, V. Kudriašov, G. Juzeliūnas, Phys. Rev. Lett. **105**, 173603 (2010).
- J. Ruseckas, V. Kudriašov, G. Juzeliūnas, R. G. Unanyan, J. Otterbach, M. Fleischhauer, Phys. Rev. A **83**, 063811 (2011).



Probe fields $\mathcal{E}_1$ and $\mathcal{E}_2$ are coupled via atomic coherences if $\langle B_1|B_2\rangle \neq 0$

Limiting cases:

- $\langle B_1 | B_2 \rangle = 0$ — two not connected Λ schemes
- $\langle B_1 | B_2 \rangle = 1$ — double Λ setup
- $0 < |\langle B_1 | B_2 \rangle| < 1$ — two connected Λ schemes



Matrix representation — Spinor slow light:

$$\mathcal{E} = \begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \end{pmatrix}, \qquad \hat{\Omega} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}, \qquad \hat{\delta} = \begin{pmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{pmatrix}$$

δ_1 and δ_2 are the detunings from two-photon resonance.

Equation for two-component probe field in the atomic cloud:

$$(c^{-1} + \hat{v}^{-1})\frac{\partial}{\partial t}\mathcal{E} + \frac{\partial}{\partial z}\mathcal{E} + \mathrm{i}\hat{v}^{-1}\hat{D}\mathcal{E} = 0$$

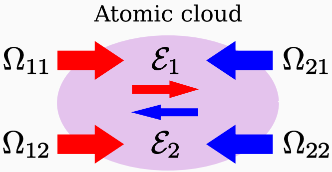
Similar to the equation for probe field in Λ scheme, only with matrices.

$\hat{D} = \hat{\Omega}\hat{\delta}\hat{\Omega}^{-1}$ is a matrix due to two-photon detuning,

$$\hat{v}^{-1} = \frac{g^2 n}{c}(\hat{\Omega}^{\dagger})^{-1}\hat{\Omega}^{-1}$$

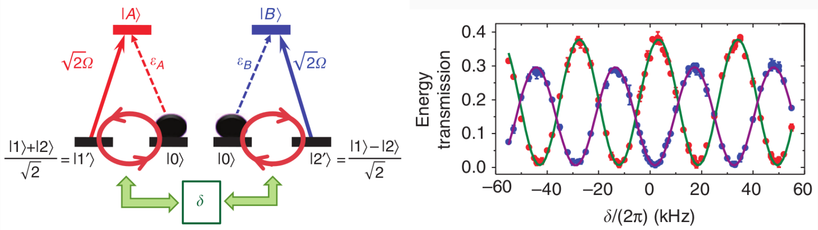
is a matrix of inverse group velocity (not necessarily diagonal).

- The group velocity is a non-diagonal matrix
- Individual probe fields do not have a definite group velocity
- Only special combinations of both probe fields (normal modes) propagate in the atomic cloud with the definite (and different) velocities
- This difference in velocities causes interference between probe fields



Two-photon detuning causes oscillations in the intensities of transmitted probe fields

M.-J. Lee, J. Ruseckas, *et al*, Nat. Commun. **5**, 5542 (2014).



- Detuning can be caused by the interaction
- For example: generation of correlated two-photon states due interaction between Rydberg atoms

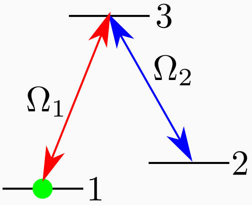
J. Ruseckas, I. A. Yu, G. Juzeliūnas, Phys. Rev. A **95**, 023807 (2017).

Equations for the atomic probability amplitudes

$$
\begin{aligned}
\mathrm{i}\partial_t\psi_1 &= -\frac{1}{2}\delta\psi_1 - \frac{1}{2}\Omega_1^*\psi_3 \\
\mathrm{i}\partial_t\psi_2 &= \frac{1}{2}\delta\psi_2 - \frac{1}{2}\Omega_2^*\psi_3 \\
\mathrm{i}\partial_t\psi_3 &= -\frac{\mathrm{i}}{2}\Gamma\psi_3 - \frac{1}{2}\Omega_1\psi_1 - \frac{1}{2}\Omega_2\psi_2
\end{aligned}
$$

Rabi frequencies of the laser fields obey the propagation equations

$$
\begin{aligned}
\partial_t\Omega_1 + c\partial_z\Omega_1 &= \frac{\mathrm{i}}{2}g\psi_3\psi_1^* \\
\partial_t\Omega_2 + c\partial_z\Omega_2 &= \frac{\mathrm{i}}{2}g\psi_3\psi_2^*
\end{aligned}
$$



Coupled and uncoupled states

$$\psi_\mathrm{C} = \frac{1}{\Omega}(\Omega_1\psi_1 + \Omega_2\psi_2)\,, \qquad \psi_\mathrm{U} = \frac{1}{\Omega}(\Omega_2^*\psi_1 - \Omega_1^*\psi_2)$$

where $\Omega = \sqrt{|\Omega_1|^2 + |\Omega_2|^2}$. Then

$$\begin{aligned} \mathrm{i}\partial_t\psi_\mathrm{U} =& \Delta\psi_\mathrm{U} + \Omega_-^*\psi_\mathrm{C} \\ \mathrm{i}\partial_t\psi_\mathrm{C} =& -\Delta\psi_\mathrm{C} + \Omega_-\psi_\mathrm{U} - \frac{1}{2}\Omega\psi_3 \\ \mathrm{i}\partial_t\psi_3 =& -\frac{\mathrm{i}}{2}\Gamma\psi_3 - \frac{1}{2}\Omega\psi_\mathrm{C} \end{aligned}$$

with

$$\begin{aligned} \Delta &= \mathrm{i}\frac{\Omega_1}{\Omega}\partial_t\frac{\Omega_1^*}{\Omega} + \mathrm{i}\frac{\Omega_2}{\Omega}\partial_t\frac{\Omega_2^*}{\Omega} + \frac{1}{2}\delta\frac{|\Omega_1|^2 - |\Omega_2|^2}{\Omega^2} \\ \Omega_- &= \mathrm{i}\frac{\Omega_2}{\Omega}\partial_t\frac{\Omega_1}{\Omega} - \mathrm{i}\frac{\Omega_1}{\Omega}\partial_t\frac{\Omega_2}{\Omega} - \delta\frac{\Omega_1}{\Omega}\frac{\Omega_2}{\Omega} \end{aligned}$$

Assuming that Ω is large: $|(\partial_t - i\Delta)\psi_{\mathrm{C}}| \ll \Omega|\psi_{\mathrm{C}}|$, $|\Omega_-| \ll \Omega$; the duration of the propagation τ_{prop} is much smaller than the life time of the adiabatons, $\Gamma\frac{|\Omega_-|^2}{\Omega^2}\tau_{\mathrm{prop}} \ll 1$:

$$
\begin{aligned}
\partial_t\Omega_1 + c\partial_z\Omega_1 =& \mathrm{i}g\frac{\Omega_-}{\Omega}\frac{\Omega_2^*}{\Omega} \\
\partial_t\Omega_2 + c\partial_z\Omega_2 =& -\mathrm{i}g\frac{\Omega_-}{\Omega}\frac{\Omega_1^*}{\Omega}
\end{aligned}
$$

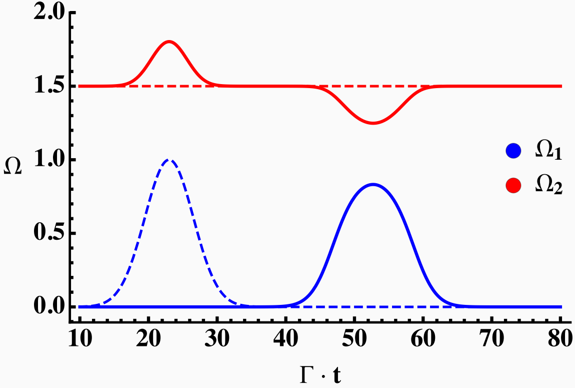
Consequences:

$$
\partial_t\Omega + c\partial_z\Omega = 0
$$

the ratio $\chi = \Omega_1/\Omega_2$ obeys the equation

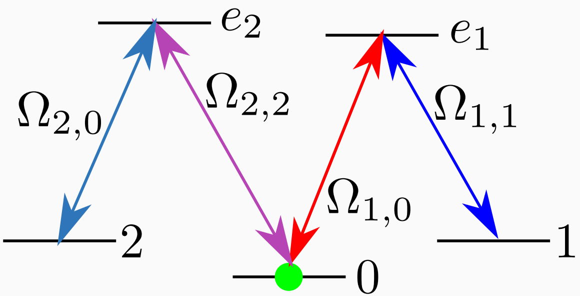
$$
\left(c^{-1} + \frac{g}{c\Omega^2}\right)\partial_t\chi + \partial_z\chi + \mathrm{i}\delta\frac{g}{c\Omega^2}\chi = 0
$$

Adiabatons in Λ system



Evolution of a pulse pair.

Adiabatic pulse propagation in multi-level systems



Uncoupled state

$$\psi_{\mathrm{U}} = \frac{1}{N_0}(\psi_0 - \chi_1^* \psi_1 - \chi_2^* \psi_2)$$

where $\chi_j = \frac{\Omega_{j,0}}{\Omega_{j,j}}$ and $N_0 = \sqrt{1 + |\chi_1|^2 + |\chi_2|^2}$

Assuming that $\Omega_j \equiv \sqrt{|\Omega_{j,j}|^2 + |\Omega_{j,0}|^2}$ are large, we get

$$(\partial_t + c\partial_z)\Omega_{j,0} = \mathrm{i}\frac{g}{N_0} W_j$$
$$(\partial_t + c\partial_z)\Omega_{j,j} = -\mathrm{i}\frac{g}{N_0}\chi_j^* W_j$$

where

$$W_1 = \frac{1}{\Omega_{1,1}^* N_0^3}[N_2^2(\mathrm{i}\partial_t - \delta_1)\chi_1 - \chi_1\chi_2^*(\mathrm{i}\partial_t - \delta_2)\chi_2]$$
$$W_2 = \frac{1}{\Omega_{2,2}^* N_0^3}[N_1^2(\mathrm{i}\partial_t - \delta_1)\chi_2 - \chi_2\chi_1^*(\mathrm{i}\partial_t - \delta_2)\chi_1]$$

and $N_j = \sqrt{1 + |\chi_j|^2}$

Conservation of energy in each subsystem separately:

$$(\partial_t + c\partial_z)\Omega_j = 0$$

Equatios for the ratios χ_j can be written in the matrix form

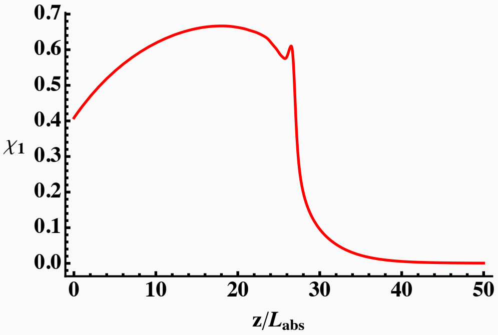
$$(c^{-1} + \hat{v}^{-1})\partial_t \chi + \partial_z \chi + \mathrm{i}\hat{v}^{-1}\hat{\delta}\chi = 0$$

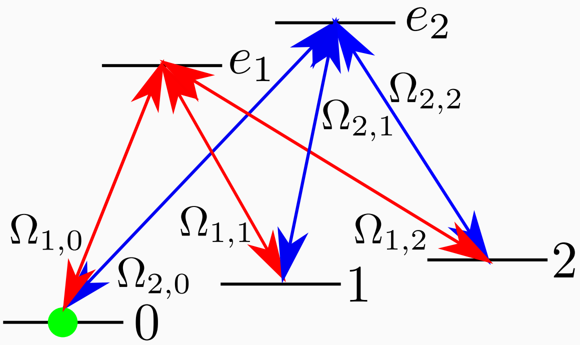
where

$$\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}, \qquad \hat{v}^{-1} = \frac{g}{c} \begin{pmatrix} \frac{1}{\Omega_1^2}\frac{N_1^4}{N_0^4} & 0 \\ 0 & \frac{1}{\Omega_2^2}\frac{N_2^4}{N_0^4} \end{pmatrix} (N_0^2 - \chi\chi^\dagger)$$

Breaking down of adiabatic approximation

- Group velocity depends on the amplitudes of the fields
- Different part of the pulse propagate with different velocity
- Region of the pulse with large steepness appears
- After some time adiabatic approximation becomes not valid





Uncoupled state

$$\psi_U = \frac{1}{N_0}\begin{vmatrix} \psi_0 & \psi_1 & \psi_2 \\ \Omega_{1,0}^* & \Omega_{1,1}^* & \Omega_{1,2}^* \\ \Omega_{2,0}^* & \Omega_{2,1}^* & \Omega_{2,2}^* \end{vmatrix} \equiv \sum_{l=0}^{2} A_l^* \psi_l$$

Assuming that $\Omega_j = \sqrt{\sum_{l=0}^{2} |\Omega_{j,l}|^2}$ are large, we get

$$(c^{-1}\partial_t + \partial_z)\hat{\Omega}_l + A_l^* \hat{v}^{-1} \sum_{n=0}^{2} A_n(\partial_t - i\delta_n)\hat{\Omega}_n = 0$$

where

$$\hat{\Omega}_l = \begin{pmatrix} \Omega_{1,l} \\ \Omega_{2,l} \end{pmatrix}$$

and

$$\hat{v}^{-1} = \frac{g}{c}\frac{1}{N_0^2}\begin{pmatrix} \Omega_2^2 & -\sum_{m=0}^{2} \Omega_{2,m}^* \Omega_{1,m} \\ -\sum_{m=0}^{2} \Omega_{1,m}^* \Omega_{2,m} & \Omega_1^2 \end{pmatrix}$$

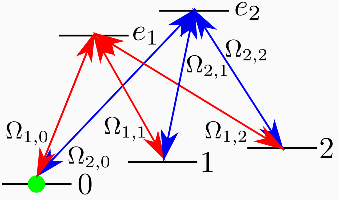
The coefficients A_l have the property

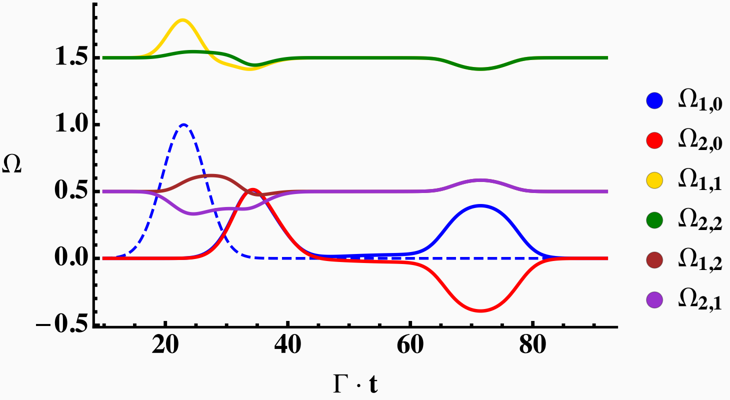
$$\sum_{l=0}^{2} A_l \Omega_{j,l} = 0$$

As a consequence

$$(\partial_t + c\partial_z)\Omega_j = 0$$

Conservation of energy in each subsystem separately





Incident on the atom cloud $\Omega_{2,2} = \Omega_{1,1}$, $\Omega_{2,1} = \Omega_{1,2}$; $\Omega_{2,0} = 0$.

Summary

Summary

- Adiabatic propagation of light pulses is not possible in all multi-level systems that have an uncoupled (dark) state.
- For example, such propagation is not possible in M-type atom-light coupling scheme.
- However, in double-tripod system adiabatic propagation of light pulses is possible
- In double-tripod system there are two different configurations of the fields that propagate without changing shape.

Thank you for your attention!