Nonlinear quantum optics for spinor slow light

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   - Two-photon states due to interactions between Rydberg atoms
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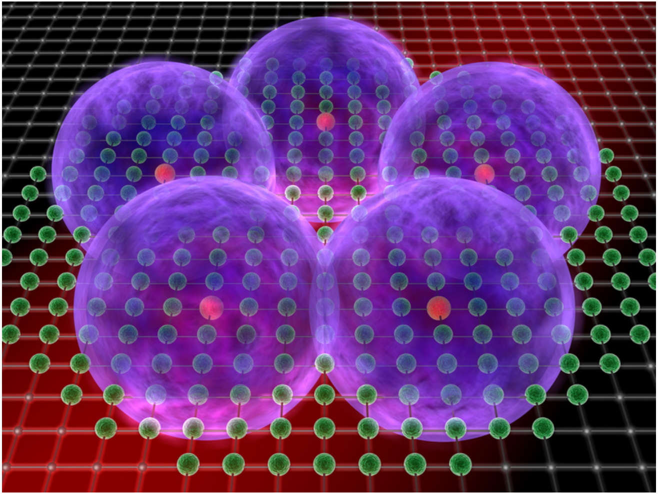
Introduction

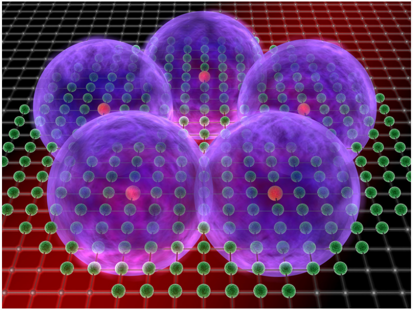
Rydberg atoms

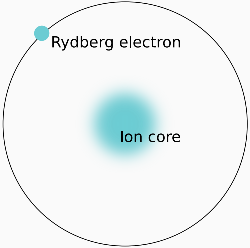


P. Schauß *et al*, Nature **491**, 87 (2012).

Rydberg atom

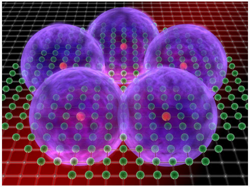
A Rydberg atom is an excited atom with an electron in a state with a very high principal quantum number $n \gtrsim 50$.





Distinctive properties of Rydberg states:

- an enhanced response to electric and magnetic field
- long decay times
- electron wavepackets move along classical orbits
- excited electron experiences Coulomb electric potential
- radius of an orbit scales as n^2
- energy level spacing decreases as $1/n^3$



Interactions between Rydberg atoms

- Transition dipole moment to nearby states scales as n^2
- Strong dipole-dipole interactions
- The interaction strength rapidly increases with n;
- The strength of interactions for $n \gtrsim 100$ can be comparable to the strength of the Coulomb interaction between ions.
- Can be used for engineering of desired many-particle states.

Dipole blockade

- If one atom is excited into the Rydberg state
  - strong interaction shifts the resonance frequencies of all the surrounding atoms
  - suppressing their excitation.
- Rydberg blockade can be applied in
  - quantum information processing
  - non-linear quantum optics using Rydberg EIT

Slow light


18 February 1999
International weekly journal of science
nature
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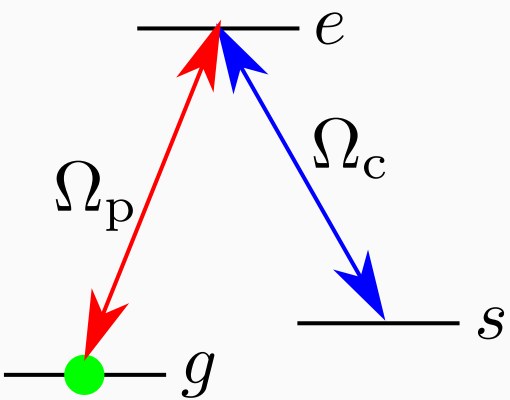
Cycling at the speed of light?



Probe beam: $\Omega_{\mathrm{p}} = \mu_{ge} E_{\mathrm{p}}$

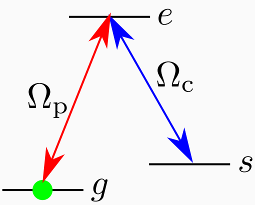
Control beam: $\Omega_{\mathrm{c}} = \mu_{ge} E_{\mathrm{c}}$

Three level Λ system

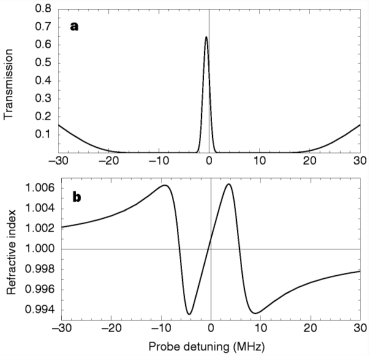
- Dark state

$$|D\rangle \sim \Omega_c|g\rangle - \Omega_p|s\rangle$$

- Transitions $g \to e$ and $s \to e$ interfere destructively
- Cancelation of absorbtion
- Electromagnetically induced transparency—EIT
- Very fragile
- Very narrow transparency window



Slow light



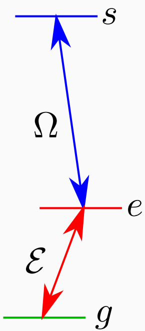
- Narrow transparency window $\Delta\omega \sim 1\,\mathrm{MHz}$
- Very dispersive medium
- Small group velocity — slow light

Rydberg EIT

- EIT $\rightarrow$ atom-light interactions without absorption
- Rydberg states $\rightarrow$ strong long-range atom-atom interactions
- As a result $\rightarrow$ photon-photon interactions.

For a single incident probe photon

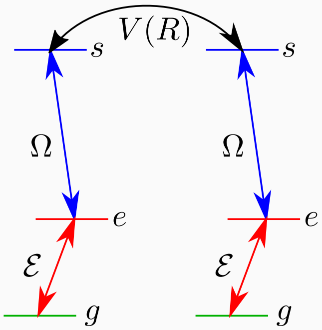
- the control field induces a transparency in a narrow spectral window via EIT
- probe photon is coupled to Rydberg excitation forming a combined quasiparticle — Rydberg polariton
- Rydberg polariton propagates at a reduced speed $\ll c$



Rydberg EIT

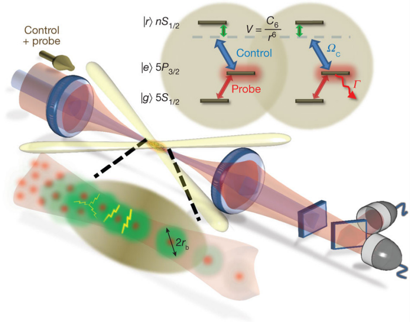
When two probe photons propagate in the Rydberg medium

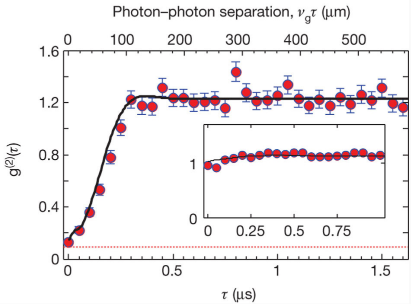
- strong interaction between two Rydberg atoms tunes the transition out of the resonance
- destroying the transparency and leading to absorption.



A. V. Gorshkov *et al*, Phys. Rev. Lett. **107**, 133602 (2011).

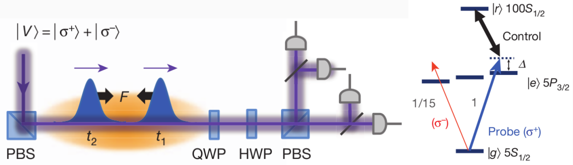
T. Peyronel *et al*, Nature **488**, 57 (2012).

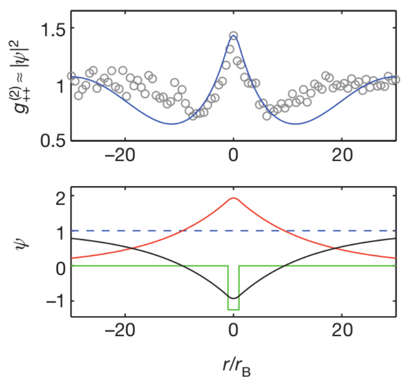




$46 \leq n \leq 100$

O. Firstenberg *et al*, Nature **502**, 71 (2013).



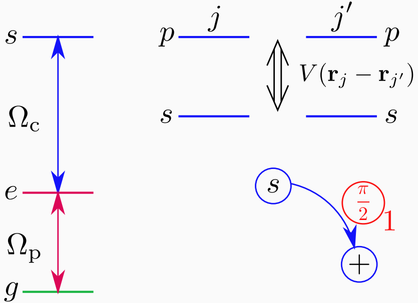


Storing slow light using two Rydberg states

J. Ruseckas, I. A. Yu, G. Juzeliūnas, Phys. Rev. A **95**, 023807 (2017).

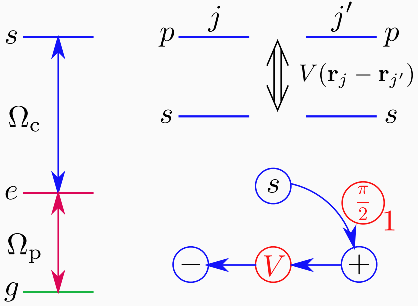
- Ladder scheme with the Rydberg state s
- Storing procedure:
    1. Probe field is stored in a coherence between ground state g and Rydberg state s
    2. $\pi/2$ pulse is applied converting the Rydberg state $|s\rangle$ to a supperposition of s and p Rydberg states

$$|+\rangle = \frac{1}{\sqrt{2}}(|s\rangle + |p\rangle)$$

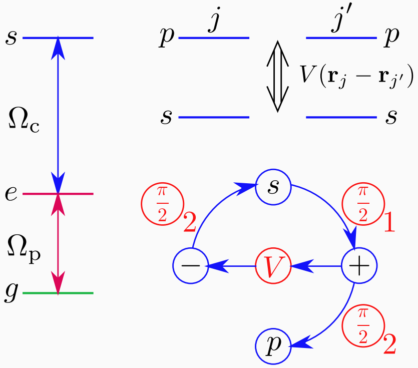


- Resonance dipole-dipole interaction between Rydberg atoms V
- Exchange of the s and p Rydberg states.
- During the storage correlated pairs of atoms are created in the initially not populated state

$$|-\rangle = \frac{1}{\sqrt{2}}(|s\rangle - |p\rangle)$$



- At the end of the sorage a second $\pi/2$ pulse is applied, converting the state $|-\rangle$ into Rydberg state $|s\rangle$ and state $|+\rangle$ into state $|p\rangle$.
- Excitations in the s state are converted into the probe photons,
- p state excitations remain in the medium.

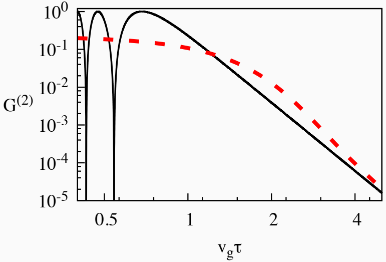


Consequences

- No regenerated slow light without interaction between the atoms
- Restored probe beam contains correlated pairs of photons

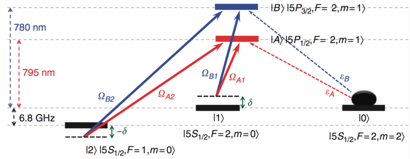
$$g^{(2)}_{\text{out}}(\tau) \sim [V(v_{g0}\tau)\,T]^2$$

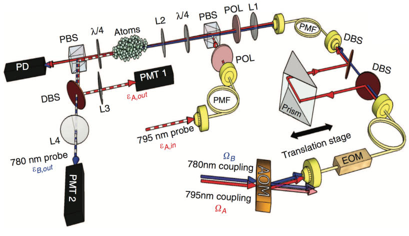
- Allows to measure interaction potential
- Corrections due to the finite spectral width of EIT (see red dashed curve)



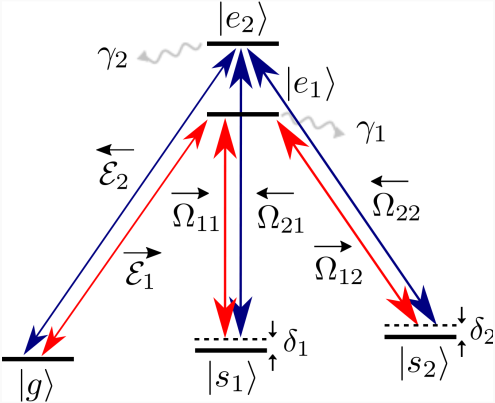
Spinor slow light

M.-J. Lee, J. Ruseckas, *et al*, Nat. Commun. **5**, 5542 (2014).

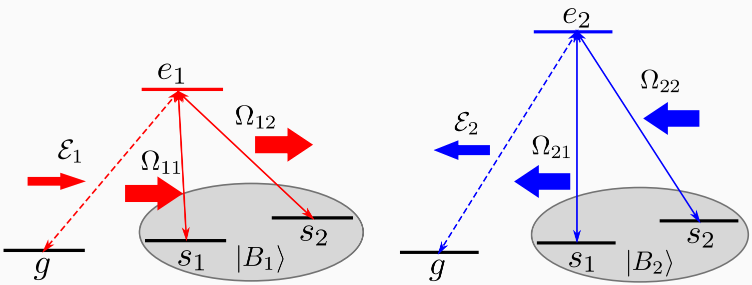




Double tripod setup



- R. G. Unanyan, J. Otterbach, M. Fleischhauer, J. Ruseckas, V. Kudriašov, G. Juzeliūnas, Phys. Rev. Lett. **105**, 173603 (2010).
- J. Ruseckas, V. Kudriašov, G. Juzeliūnas, R. G. Unanyan, J. Otterbach, M. Fleischhauer, Phys. Rev. A **83**, 063811 (2011).



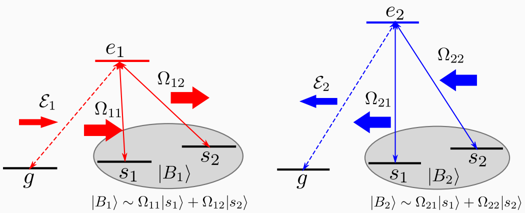
$|B_1\rangle \sim \Omega_{11}|s_1\rangle + \Omega_{12}|s_2\rangle$ $|B_2\rangle \sim \Omega_{21}|s_1\rangle + \Omega_{22}|s_2\rangle$

Probe fields $\mathcal{E}_1$ and $\mathcal{E}_2$ are coupled via atomic coherences if $\langle B_1|B_2\rangle \neq 0$

Double tripod setup

Limiting cases:

- $\langle B_1|B_2\rangle = 0$ — two not connected Λ schemes
- $\langle B_1|B_2\rangle = 1$ — double Λ setup
- $0 < |\langle B_1|B_2\rangle| < 1$ — two connected Λ schemes



Matrix representation — Spinor slow light:

$$\mathcal{E} = \begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \end{pmatrix}, \qquad \hat{\Omega} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}, \qquad \hat{\delta} = \begin{pmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{pmatrix}$$

δ_1 and δ_2 are the detunings from two-photon resonance.

Equation for two-component probe field in the atomic cloud:

$$(c^{-1} + \hat{v}^{-1})\frac{\partial}{\partial t}\mathcal{E} + \frac{\partial}{\partial z}\mathcal{E} + \mathrm{i}\hat{v}^{-1}\hat{D}\mathcal{E} = 0$$

Similar to the equation for probe field in Λ scheme, only with matrices.

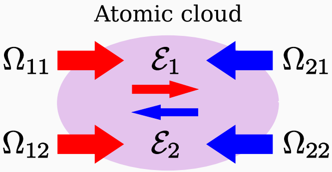
$\hat{D} = \hat{\Omega}\hat{\delta}\hat{\Omega}^{-1}$ is a matrix due to two-photon detuning,

$$\hat{v}^{-1} = \frac{g^2 n}{c}(\hat{\Omega}^{\dagger})^{-1}\hat{\Omega}^{-1}$$

is a matrix of inverse group velocity (not necessarily diagonal).

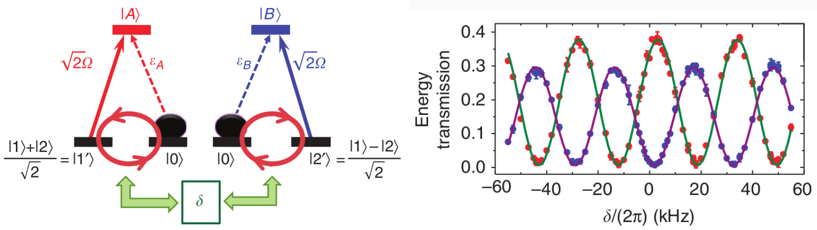
Spinor slow light

- The group velocity is a non-diagonal matrix
- Individual probe fields do not have a definite group velocity
- Only special combinations of both probe fields (normal modes) propagate in the atomic cloud with the definite (and different) velocities
- This difference in velocities causes interference between probe fields



Two-photon detuning causes oscillations in the intensities of transmitted probe fields

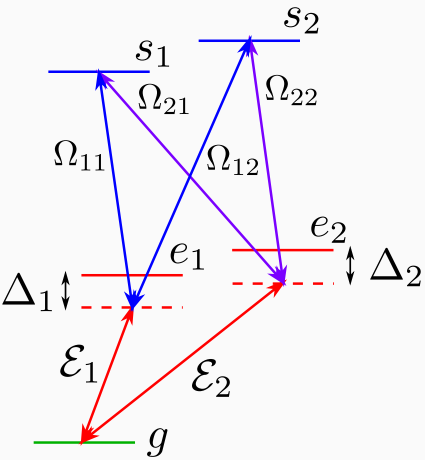
M.-J. Lee, J. Ruseckas, *et al*, Nat. Commun. **5**, 5542 (2014).



- Detuning can be caused by the interaction
- For example: generation of correlated two-photon states due interaction between Rydberg atoms

J. Ruseckas, I. A. Yu, G. Juzeliūnas, Phys. Rev. A **95**, 023807 (2017).

Nonlinear quantum optics for spinor slow light



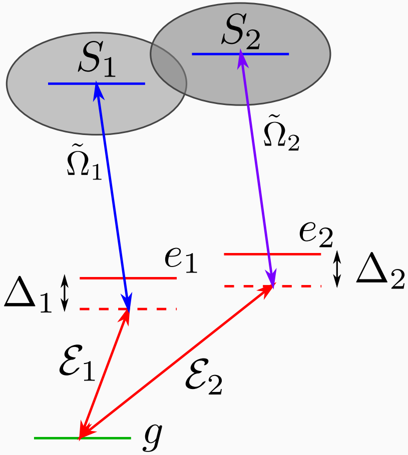
Double tripod atom-light coupling scheme involving the Rydberg levels s_1 and s_2.

In the continuum approximation the probe fields and atomic excitations can be represented by slowly varying operators $\hat{\mathcal{E}}_j^\dagger(z)$, $\hat{\Psi}_{e_j}^\dagger(z)$ and $\hat{\Psi}_{s_j}^\dagger(z)$.

The probe fields are assumed to be sufficiently weak at the input, so that the contribution due to more than two photons is not important

Two-excitation wave functions

$$
\begin{aligned}
\Phi_{\mathcal{E}_j\mathcal{E}_l}(z, z', t) =& \langle \mathrm{vac}|\hat{\mathcal{E}}_j(z, t)\hat{\mathcal{E}}_l(z', t)|\Phi\rangle \\
\Phi_{\mathcal{E}_j s_l}(z, z', t) =& \langle \mathrm{vac}|\hat{\mathcal{E}}_j(z, t)\hat{\Psi}_{s_l}(z', t)|\Phi\rangle \\
\Phi_{s_j s_l}(z, z', t) =& \langle \mathrm{vac}|\hat{\Psi}_{s_j}(z, t)\hat{\Psi}_{s_l}(z', t)|\Phi\rangle
\end{aligned}
$$



$$|S_1\rangle \sim \Omega_{11}|s_1\rangle + \Omega_{12}|s_2\rangle\,, \qquad |S_2\rangle \sim \Omega_{21}|s_1\rangle + \Omega_{22}|s_2\rangle$$

Two-photon wave funtion

$$\Phi_{\mathcal{E}_j\mathcal{E}_l} = -\frac{1}{2}(\Phi_{\mathcal{E}_jS_l} + \Phi_{S_j\mathcal{E}_l})$$

One-photon wave functions

$$c\partial_z\Phi_{\mathcal{E}_jS_l} = \frac{\mathrm{i}}{2}g^2\tilde{\Delta}_j^{-1}(\Phi_{\mathcal{E}_jS_l} + \Phi_{S_jS_l})$$

$$c\partial_{z'}\Phi_{S_j\mathcal{E}_l} = \frac{\mathrm{i}}{2}g^2\tilde{\Delta}_l^{-1}(\Phi_{S_j\mathcal{E}_l} + \Phi_{S_jS_l})$$

Two atomic excitations

$$\sum_m \tilde{\Delta}_m^{-1}\left(v_{j,m}(\Phi_{\mathcal{E}_mS_l} + \Phi_{S_mS_l}) + v_{l,m}(\Phi_{S_j\mathcal{E}_m} + \Phi_{S_jS_m})\right) - \frac{2c}{g^2}V(z' - z)\Phi_{S_jS_l} = 0$$

Closed equation

$$\begin{aligned} \mathrm{i}\partial_R \Phi_{\mathcal{E}_j\mathcal{E}_l} &= -4L_{\mathrm{abs}}\frac{\tilde{\Delta}}{\Gamma}\partial_r^2\Phi_{\mathcal{E}_j\mathcal{E}_l} \\ &\quad + \frac{\mathrm{i}}{\bar{v} - L_{\mathrm{abs}}\frac{\tilde{\Delta}}{\Gamma}V(r)}\sum_m (v_{l,m}\partial_r\Phi_{\mathcal{E}_j\mathcal{E}_m} - v_{j,m}\partial_r\Phi_{\mathcal{E}_m\mathcal{E}_l}) \\ &\quad + \frac{V(r)}{\bar{v} - L_{\mathrm{abs}}\frac{\tilde{\Delta}}{\Gamma}V(r)}\sum_{m,n} A_{jl,mn}\Phi_{\mathcal{E}_m\mathcal{E}_n} \end{aligned}$$

Here

$$R = \frac{1}{2}(z + z'), \qquad r = z - z', \qquad \bar{v} = \frac{1}{2}(v_{1,1} + v_{2,2})$$

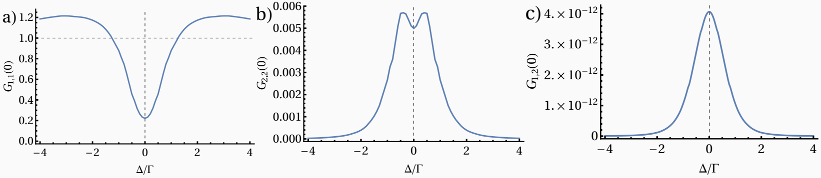
Equal one-photon detunings: consequences

- When $\Delta \gg \Gamma$, equation of propagation has the form of a Schrödinger equation; the center of mass coordinate R plays the role of time.
    - The first term: represents the kinetic energy
    - The second term: couples the linear momentum and represents spin-orbit coupling for the photons.
    - The last term: effective potential for the photons $\mathcal{V}(r) = \Gamma/(2L_{\mathrm{abs}}\Delta)$ when r is smaller than the blockade radius

Equal one-photon detunings: consequences

- When $\Delta \ll \Gamma$, the propagation equation acquires the form of a diffusion equation.
    - The diffusion term: spreading out of the wave packet of slow light caused by the non-adiabatic losses due to the deviation from the EIT central frequency.
    - The last term: the absorption of the photons when the relative distance is small; the Rydberg blockade effect.

Second-order correlation functions

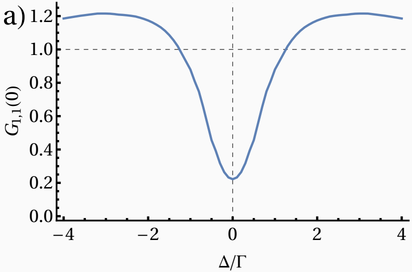


Only the first probe beam with the amplitude a is incident on the atom cloud; $v_{1,2}/v_{1,1} = 1/2$.

Second-order correlation functions normalized to the intensity of the incident probe beam

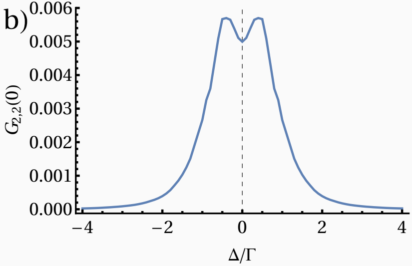
$$G^{(2)}_{j,l}(0) = \frac{1}{a^4}|\Phi_{\mathcal{E}_j\mathcal{E}_l}(R = L, r = 0)|^2$$

Photon bunching, $G^{(2)}_{1,1}(0) > 1$, due to the atom-atom interactions



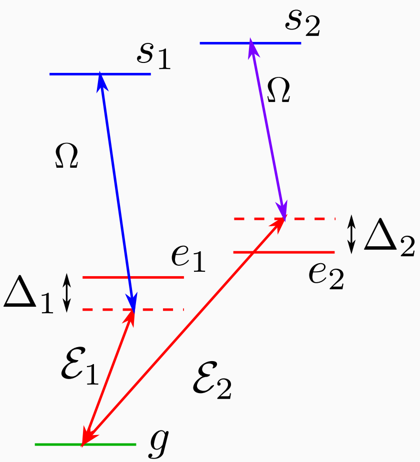
Photons are transferred from the first to the second probe beam due to

- atom-atom interactions
- non-diagonal elements of the group velocity matrix



Opposite signs of one-photon detunings

- In contrast to a single ladder scheme, in the double tripod setup the two one-photon detunings can take different values
- Problem: the approximations leading to the single closed equation are not valid when detunings are different.



No transfer of photons between probe beams.

One needs to solve two coupled equations

$$c\partial_z \Phi_{\mathcal{E}_j s_l} = \frac{\mathrm{i}}{2} g^2 \tilde{\Delta}_j^{-1} (\Phi_{\mathcal{E}_j s_l} + \Phi_{s_j s_l})$$
$$c\partial_{z'} \Phi_{s_j \mathcal{E}_l} = \frac{\mathrm{i}}{2} g^2 \tilde{\Delta}_l^{-1} (\Phi_{s_j \mathcal{E}_l} + \Phi_{s_j s_l})$$

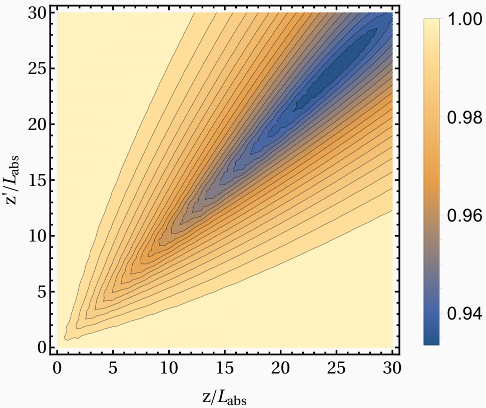
where

$$\Phi_{s_j s_l} = \frac{\tilde{\Delta}_j^{-1} \Phi_{\mathcal{E}_j s_l} + \tilde{\Delta}_l^{-1} \Phi_{s_j \mathcal{E}_l}}{\frac{2c}{g^2 v} V(z' - z) - (\tilde{\Delta}_j^{-1} + \tilde{\Delta}_l^{-1})}$$

Two-photon wave funtion

$$\Phi_{\mathcal{E}_j \mathcal{E}_l} = -\frac{1}{2} (\Phi_{\mathcal{E}_j s_l} + \Phi_{s_j \mathcal{E}_l})$$

Both probe beams incident on the atom cloud



Two-photon wave function when each incident probe beam contains a single photon. Absolute value of one-photon detuning $\Delta/\Gamma = 2.5$

Summary

Summary

- The double tripod scheme can combine spin-orbit coupling for the spinor slow light with an interaction between photons.
- Atom-atom interactions can cause transfer of photons between probe beams.
- In contrast to a single ladder scheme, in the double tripod setup the two one-photon detunings can take different values.
- Large one-photon detunings lead to an effective interaction between photons which is attractive when detunings are equal and repulsive when they have opposite signs.

Thank you for your attention!