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COLD ATOMS AND BOSE–EINSTEIN CONDENSATE

Manipulation of Slow Light with Orbital Angular Momentum in Cold Atomic Gases[1]

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Abstract—We study the propagation of a weak pulse of slow light in a cloud of cold atoms controlled by two additional laser beams of larger intensity in a tripod configuration of the light-matter coupling. We consider a case where one of the control beams has an optical vortex and thus has a zero intensity at the center. The presence of the second control beams restores adiabaticity in the propagation of the probe beam. This makes it possible to exchange the optical vortex between the control and probe fields during the storage. We analyze conditions for the vortex of the control beam to be transferred efficiently to the restored probe beam.

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I. INTRODUCTION

Light can be slowed down by seven orders of magnitude using the electromagnetically induced transparency (EIT) [1]. The EIT makes a resonant, opaque medium transparent by means of quantum interference between the optical transitions induced by the control and probe laser fields. As a result, a weak pulse of probe light travels slowly and almost without a dissipation in a resonant medium controlled by another laser beam [1–6]. EIT was shown not only to slow down dramatically light pulses [1] but also to store them [7] in atomic gases. Following the proposal of [8], storage and release of a probe pulse has been demonstrated [7] by dynamically changing the intensity of the control laser. The possibility to coherently control the propagation of quantum light pulses opens up interesting applications such as generation of nonclassical states in atomic ensembles and reversible quantum memories for slow light [5, 6, 8, 9]. Furthermore, propagation of slow light through moving media (linearly [10, 11] or rotationally [12–14]) may be used for the light memories and rotational sensing.

An interesting issue is propagation of the slow light carrying an orbital angular momentum (OAM) [14]. The OAM of light [15] represents a new degree of freedom which can be exploited in the quantum computation and quantum information storage. Here we demonstrate how the storage of the slow light can be manipulated using control laser beams with and without the OAM. We study the propagation of a weak pulse of slow light in a cloud of cold atoms controlled by two additional laser beams of larger intensity. In that case the probe and two control fields induce transitions between the atomic energy levels in a tripod configuration of the light-atom coupling [16–18]. We show that the regenerated slow light can acquire an OAM by applying a control beam with the OAM. The application of the second control beam without the OAM quakes it possible to avoid difficulties with a violation of the adiabaticity in the slow light dynamics due to the zero intensity of the control vortex beam at the center.

In the following we derive and analyze a general equation for the propagation of the probe beam in an ensemble of tripod atoms without applying the slowly-varying amplitude approximation [18]. This makes it possible to consider situations where control and/or probe beams have the OAM. The atoms in our model are cold enough to be treated as quantum fields. We choose the atoms to be bosons populating a single center of mass single state to form a Bose–Einstein condensate.

II. FORMULATION

Consider an ensemble of tripod-type atoms characterized by three hyperfine ground levels 1, 2, and 3, as well as an electronic excited level 0 (Fig. 1). The translational motion of atoms is represented by a four component column operator $\Psi(\mathbf{r})$, where the components $\Psi_1 \equiv \Psi_1(\mathbf{r}, t)$, $\Psi_2 \equiv \Psi_2(\mathbf{r}, t)$, $\Psi_3 \equiv \Psi_3(\mathbf{r}, t)$, and $\Psi_0 \equiv \Psi_0(\mathbf{r}, t)$, are the field operators describing to the center of mass motion in the four internal atomic states. The quantum nature of the atoms comprising the medium will determine whether these field operators obey Bose–Einstein or Fermi–Dirac commutation relations. The atoms interact with three light fields in a tripod configuration of the atom-light coupling [16–18]. Specifically, two strong classical control lasers induce transitions $|2\rangle \longrightarrow |0\rangle$, $|3\rangle \longrightarrow |0\rangle$

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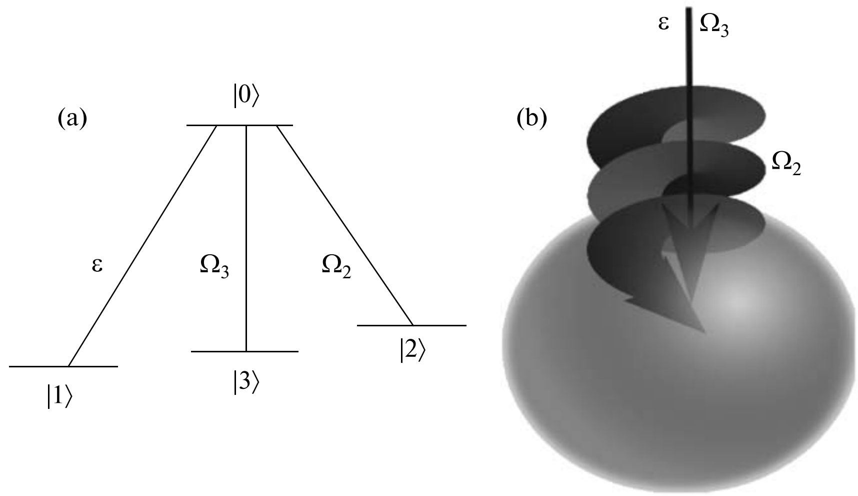

Fig. 1. Light induced transitions in the cloud of cold atoms involving a tripod configuration of the internal atomic states.

and a weaker quantum probe field drives a transition $|1\rangle \longrightarrow |0\rangle$, see Fig. 1.

A. Equation for the Probe Field

The electric field of the probe beam is

$$\mathbf{E}(\mathbf{r}, t) = \hat{\mathbf{e}}\sqrt{\frac{\hbar\omega}{2\varepsilon_0}}\mathcal{E}(\mathbf{r}, t)e^{-i\omega t} + \text{h.c.}, \quad (1)$$

where $\omega = ck$ is the central frequency of the probe photons, $\mathbf{k} = \hat{\mathbf{z}}k$ is the wave vector, and $\hat{\mathbf{e}} \perp \hat{\mathbf{z}}$ is the unit polarization vector. This field is considered to be either a classical variable or a quantum operator. We have chosen the dimensions of the electric field amplitude S so that its squared modulus represents the number density of probe photons.

The probe field $\mathbf{E}(\mathbf{r}, t)$ obeys the following wave equation:

$$c^2\nabla^2\mathbf{E} - \frac{\partial^2\mathbf{E}}{\partial t^2} = \frac{1}{\varepsilon_0}\frac{\partial^2\mathbf{P}}{\partial t^2}, \quad (2)$$

where $\mathbf{P} = \hat{\mathbf{e}}\mu\Psi_1^\dagger\Psi_0 + \text{h.c.}$ is the polarization field of atoms, μ being the dipole moment of the atomic transition $|1\rangle \longrightarrow |0\rangle$. Let us introduce the slowly varying matter field operators

$$\Phi_1 = \Psi_1 e^{i\omega_1 t}, \quad (3)$$

$$\Phi_2 = \Psi_2 e^{i(\omega_1 + \omega - \omega_{c2})t}, \quad (4)$$

$$\Phi_3 = \Psi_3 e^{i(\omega_1 + \omega - \omega_{c3})t}, \quad (5)$$

$$\Phi_0 = \Psi_0 e^{i(\omega_1 + \omega)t}, \quad (6)$$

with $\hbar\omega_1$ being the energy of the atomic ground state 1, and ω_{c2}, ω_{c3} being the frequencies of the control fields.

The probe field is quasi-monochromatic, so its amplitude $\mathcal{E} \equiv \mathcal{E}(\mathbf{r}, t)$ changes little over the time of an optical cycle. In this case, the following equation holds for the slowly varying amplitude of the probe field:

$$\left(\frac{\partial}{\partial t} - i\frac{c^2}{2\omega}\nabla^2 - i\frac{\omega}{2}\right)\mathcal{E} = ig\Phi_1^\dagger\Phi_0, \quad (7)$$

where the parameter $g = \mu\sqrt{\omega/2\varepsilon_0\hbar}$ characterizes the strength of coupling of the probe field with the atoms. Note that, unlike in the usual treatment of slow light, we have retained the second-order derivative ∇^2 in the equation of motion (7). This allows us to account for the fast changes of $\mathcal{E}$ in a direction perpendicular to the wave vector $\mathbf{k}$, i.e., in the xy plane. Therefore, our analysis can be applied to the twisted beams of light $\mathcal{E}(\mathbf{r}, t) \sim \exp(il\varphi)$ carrying an OAM $\hbar l$ per photon.

B. Equations for the Matter Field Operators

In the following we shall adapt a semi-classical picture in which both electromagnetic and matter field operators are replaced by c numbers. The equations for the matter fields are then

$$\hat{K}\Phi_1 = V_1(\mathbf{r})\Phi_1 - \hbar g\mathcal{E}^*\Phi_0, \quad (8)$$

$$\hat{K}\Phi_0 = \hbar(\omega_{01} - i\gamma)\Phi_0 + V_0(\mathbf{r})\Phi_0 - \hbar\Omega_{c2}\Phi_2 - \hbar\Omega_{c3}\Phi_3 - \hbar g\mathcal{E}\Phi_1, \quad (9)$$

$$\hat{K}\Phi_2 = \hbar\omega_{21}\Phi_2 + V_2(\mathbf{r})\Phi_2 - \hbar\Omega_{c2}^*\Phi_0, \quad (10)$$

$$\hat{K}\Phi_3 = \hbar\omega_{31}\Phi_3 + V_3(\mathbf{r})\Phi_3 - \hbar\Omega_{c3}^*\Phi_0, \quad (11)$$

with

$$\hat{K} = i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\nabla^2. \quad (12)$$

Here m is the atomic mass, $V_j(\mathbf{r})$ is the trapping potential for an atom in the electronic state j ($j = 1, 2, 3, 0$), Ω_{c2} and Ω_{c3} are the Rabi frequencies of control lasers driving the transitions $|2\rangle \longrightarrow |0\rangle$ and $|3\rangle \longrightarrow |0\rangle$, γ is the decay rate of the excited level, $\omega_{21} = \omega_2 - \omega_1 + \omega_{c2} - \omega$ and $\omega_{31} = \omega_3 - \omega_1 + \omega_{c3} - \omega$ are frequencies of the electronic detuning from the two-photon resonances, $\omega_{01} = \omega_0 - \omega_1 - \omega$ is the frequency of the electronic detuning from the one-photon resonance. The terms containing atomic mass m in Eqs. (8)–(11) are important for the description of the light-dragging effects [10–14], Note also that the equation of motion (8) for Φ_1 does not explicitly accommodate collisions between the ground-state atoms. In the case where the atoms in the internal ground-state 1 form a BEC, the collisional effects can be included replacing $V_1(\mathbf{r})$ by $V_1(\mathbf{r}) + g_{11}|\Phi_1|^2$ in Eq. (8) to yield a mean-field equation for the condensate wave function Φ_1, where $g_{11} = 4\pi\hbar^2 a_{11}/m$ and a_{11} is the scattering length between the condensate atoms in the internal state 1. In Eqs. (8)–(11), the coupling of atoms with the probe and control fields has been written using the rotating wave approximation. Therefore, the last term in Eq. (8) has a positive frequency part of the probe field (i.e., $\mathcal{E}^*$), whereas the last term in Eq. (9) has a negative frequency part ($\mathcal{E}$). For the same reason, Eq. (9) contains factors Ω_{c2} and Ω_{c3}, whereas Eqs. (10) and (11) contain factors Ω_{c2}^* and Ω_{c3}^*.

III. PROPAGATION OF PROBE BEAM

A. Dark and Bright States

To analyze the atomic dynamics, it is convenient to introduce the wave-function for the atomic motion in the bright state

$$\Phi_B = \frac{1}{\Omega_c}(\Omega_{c2}\Phi_2 + \Omega_{c3}\Phi_3). \quad (13)$$

The corresponding wave-function of the atomic motion in the dark state is

$$\Phi_D = \frac{1}{\Omega_c}(\Omega_{c3}^*\Phi_2 - \Omega_{c2}^*\Phi_3), \quad (14)$$

where

$$\Omega_c = \sqrt{|\Omega_{c2}|^2 + |\Omega_{c3}|^2} \quad (15)$$

is the total Rabi frequency. It is the wave-function Φ_B which is featured in the equation of motion (9) for the excited state wave-function. On the other hand, the dark state is not coupled directly to the excited level 0. The original fields Φ_2 and Φ_3 can be expressed through Φ_B and Φ_D as

$$\Phi_2 = \frac{1}{\Omega_c}(\Omega_{c2}^*\Phi_B + \Omega_{c3}\Phi_D), \quad (16)$$

$$\Phi_3 = \frac{1}{\Omega_c}(\Omega_{c3}^*\Phi_B - \Omega_{c2}\Phi_D). \quad (17)$$

Initially the atoms are in the ground level 1 and the Rabi frequency of the probe field is considered to be much smaller than Ω_c. Consequently one can neglect the last term in Eq. (8) that causes depletion of the ground level 1, giving $\hat{K}\Phi_1 = V_1(\mathbf{r})\Phi_1$. If the atoms in the internal ground-state 1 form a BEC, its wave function $\Phi_1 = \sqrt{n}\exp(iS_1)$ represents an incident variable determined by the atomic density n and the condensate phase S_1. The latter phase will not play an important role in our subsequent analysis, since we are not interested in the effects due to the condensate dynamics.

B. Adiabatic Approximation

Suppose the control and probe beams are tuned close to the two-photon resonance. Application of such laser beams cause electromagnetically induced transparency (EIT) in which the transitions $|2\rangle \longrightarrow |0\rangle$, $|3\rangle \longrightarrow |0\rangle$, and $|1\rangle \longrightarrow |0\rangle$ interfere destructively preventing population of the excited state 0. The adiabatic approximation is obtained neglecting the excited state population in Eq. (9), giving

$$\Phi_B = -g\frac{\Phi_1}{\Omega_c}\mathcal{E}. \quad (18)$$

When the adiabatic approximation is valid, the bright and dark states are coupled weakly. Therefore, combining Eqs. (10), (11) and neglecting the dark state contribution Φ_D, we arrive at the equation for the bright state wave-function

$$\hat{K}\Phi_B = i\frac{\hbar}{m}\mathbf{A}\nabla\Phi_B + U\Phi_B - \hbar\Omega_c\Phi_0, \quad (19)$$

where

$$\mathbf{A} = i\hbar(\xi_2\nabla\xi_2^* + \xi_3\nabla\xi_3^*) \quad (20)$$

is the effective vector potential for the atoms and

$$U = -\frac{\hbar^2}{2m}(\xi_2\nabla^2\xi_2^* + \xi_3\nabla^2\xi_3^*) + (V_2 + \hbar\omega_{21})|\xi_2|^2 + (V_3 + \hbar\omega_{31})|\xi_3|^2 - i\hbar\left(\xi_2\frac{\partial}{\partial t}\xi_2^* + \xi_3\frac{\partial}{\partial t}\xi_3^*\right) \quad (21)$$

is the corresponding effective scalar potential arising due the two-photon detuning, with

$$\xi_2 = \frac{\Omega_{c2}}{\Omega_c}, \quad \xi_3 = \frac{\Omega_{c3}}{\Omega_c}. \tag{22}$$

The derivation of Eq. (19) can be found in more details in our earlier work [19] on the light-induced gauge potentials for the Λ type atoms.

Equation (19) relates Φ_0 to the bright state Φ_B as

$$\Phi_0 = \frac{1}{\hbar\Omega_c}\left[-\hat{K} + i\frac{\hbar}{m}\mathbf{A}\nabla + U\right]\Phi_B. \tag{23}$$

Finally, Eqs. (7), (18), and (23) provide a closed equation for the electric field amplitude $\mathscr{E}$:

$$\left(\frac{\partial}{\partial t} - i\frac{c^2}{2\omega}\nabla^2 - i\frac{\omega}{2}\right)\mathscr{E} = -i\frac{g^2\Phi_1^*}{\hbar\Omega_c}\left[-\hat{K} + i\frac{\hbar}{m}\mathbf{A}\nabla + U\right]\frac{\Phi_1}{\Omega_c}\mathscr{E}. \tag{24}$$

This equation applies a wide variety of phenomena. In particular, it can be used to model light storage by introducing time dependence in Ω_c or light dragging due to spatial variation of Ω_c or Φ_1.

C. Regenerating of the Probe Beam: Importance of the Non-adiabatic Terms

In order to describe releasing of the stored light, we should include non-adiabatic corrections to the equation of motion (24). This can be done in the following way. From Eq. (9), we express the bright state Φ_B

$$\Phi_B = -\frac{g\mathscr{E}}{\Omega_c}\Phi_1 + \frac{1}{\hbar\Omega_c}[-\hat{K} + V_0(\mathbf{r}) + \hbar(\omega_{01} - i\gamma)]\Phi_0. \tag{25}$$

Substituting Eq. (23) into (25), we get

$$\Phi_B = -\frac{g\mathscr{E}}{\Omega_c}\Phi_1 + \frac{1}{\hbar\Omega_c}[-\hat{K} + V_0(\mathbf{r}) + \hbar(\omega_{01} - i\gamma)] \times \frac{1}{\hbar\Omega_c}\left[-\hat{K} + i\frac{\hbar}{m}\mathbf{A}\nabla + U\right]\Phi_B. \tag{26}$$

The term with the decay rate γ is larger than other non-adiabatic corrections in the above equation giving

$$\Phi_B \approx -\frac{g\mathscr{E}}{\Omega_c}\Phi_1 - \frac{i\gamma}{\hbar\Omega_c^2}\left[-\hat{K} + i\frac{\hbar}{m}\mathbf{A}\nabla + U\right]\Phi_B. \tag{27}$$

For simplicity of the subsequent analysis, we keep only the time derivative in the above equation:

$$\Phi_B \approx -\frac{g\mathscr{E}}{\Omega_c}\Phi_1 - \frac{\gamma}{\Omega_c^2}\frac{\partial}{\partial t}\Phi_B. \tag{28}$$

The solution to this equation is

$$\Phi_B = \Phi_B(0)e^{-\frac{\Omega_c^2}{\gamma}t} - \frac{\Omega_c^2}{\gamma}e^{-\frac{\Omega_c^2}{\gamma}t}\int_0^t \frac{g\mathscr{E}}{\Omega_c}\Phi_1 e^{\frac{\Omega_c^2}{\gamma}t'}dt'. \tag{29}$$

Assuming that the control beam is switched on suddenly at $t = 0$ and then changes slowly during the characteristic relation time γ/Ω_c^2, we can perform the integration and obtain

$$\Phi_B = \Phi_B(0)e^{-\frac{\Omega_c^2}{\gamma}t} - \frac{g\mathscr{E}}{\Omega_c}\Phi_1\left(1 - e^{-\frac{\Omega_c^2}{\gamma}t}\right). \tag{30}$$

Using Eqs. (23) and (30), the equation for the electric field (7) takes the form

$$\left(\frac{\partial}{\partial t} - i\frac{c^2}{2\omega}\nabla^2 - i\frac{\omega}{2}\right)\mathscr{E} = -i\frac{g^2\Phi_1^*}{\hbar\Omega_c}\left[-\hat{K} + i\frac{\hbar}{m}\mathbf{A}\nabla + U\right]\frac{\Phi_1}{\Omega_c}\mathscr{E}\left(1 - e^{-\frac{\Omega_c^2}{\gamma}t}\right) + i\frac{g\Phi_1^*}{\hbar\Omega_c}\left[-\hat{K} + i\frac{\hbar}{m}\mathbf{A}\nabla + U\right]\Phi_B(0)e^{-\frac{\Omega_c^2}{\gamma}t}. \tag{31}$$

At $t = 0$, the probe field is off, and the information on the previously stored probe beam being contained in the atomic coherence $\Phi_B(0)$. The regeneration of the probe beam is described by the second term on the r.h.s. of Eq. (31) representing the source for the electric field. Retaining only the temporal derivatives in Eq. (31), we get the equation describing the generation of the electric field

$$\left[1 + \frac{g^2 n}{\Omega_c^2}\left(1 - e^{-\frac{\Omega_c^2}{\gamma}t}\right)\right]\frac{\partial}{\partial t}\mathscr{E} = -\frac{g\Phi_1^*}{\gamma}[g\Phi_1\mathscr{E} + \Omega_c\Phi_B(0)]e^{-\frac{\Omega_c^2}{\gamma}t}, \tag{32}$$

with the initial condition $\mathscr{E}(0) = 0$ at $t = 0$. For time in access of the relaxation time γ/Ω_c^2 the regenerated probe field evolves to a steady-state value complying with the adiabatic condition (18)

$$\mathscr{E} = -\frac{\Omega_c}{g\Phi_1}\Phi_B(0). \tag{33}$$

In this way, the regenerated electric field $\mathscr{E}$ is indeed determined by the initial atomic coherence $\Phi_B(0)$. The subsequent evolution of the probe field is described by the adiabatic equation of motion (24) containing both the temporal and spatial derivatives subject to the initial condition (33).

D. Co-propagating Control and Probe Beams

Suppose the probe and control beams co-propagate:

$$\mathcal{E}(\mathbf{r}, t) = \tilde{\mathcal{E}}(\mathbf{r}, t)e^{ikz}, \tag{34}$$

$$\Omega_{c2}(\mathbf{r}, t) = \Omega_2(\mathbf{r}, t)e^{ik_{c2}z}, \tag{35}$$

$$\Omega_{c3}(\mathbf{r}, t) = \Omega_3(\mathbf{r}, t)e^{ik_{c3}z}, \tag{36}$$

where k_{c2} and k_{c3} [are the wave numbers of the control beams. For paraxial beams the amplitudes $\tilde{\mathcal{E}}(\mathbf{r}, t)$, $\Omega_2(\mathbf{r}, t)$ and $\Omega_3(\mathbf{r}, t)$ depend weakly in the propagation direction z compared to the variation of the exponential factors. Equation (24) for the probe field takes then the form

$$\frac{\partial}{\partial t}\tilde{\mathcal{E}} + v_g\left[\frac{\partial}{\partial z} + \left(\frac{1}{v_g} - \frac{1}{c}\right)i\delta - i\frac{1}{2k}\nabla_\perp^2\right]\tilde{\mathcal{E}} = \left(1 - \frac{v_g}{c}\right)\frac{1}{\Omega_c}\frac{\partial\Omega_c}{\partial t}\tilde{\mathcal{E}}, \tag{37}$$

where we have replaced ∇^2 its transverse part $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ because of the paraxial approximation. Here

$$v_g = \frac{c}{1 + g^2 n/\Omega_c^2} \tag{38}$$

is the radiative group velocity and

$$\delta = \left(\frac{1}{\hbar}V_2 + \omega_{21}\right)|\xi_2|^2 + \left(\frac{1}{\hbar}V_3 + \omega_{31}\right)|\xi_3|^2 - \frac{1}{\hbar}V_1 \tag{39}$$

is the two-photon frequency mismatch. In writing Eq. (37), we have neglected the terms containing atomic mass m, since from now on we are not interested in the effects due to the atomic motion. The term with spatial derivative $\partial/\partial z$ in Eq. (37) describes the radiative propagation along the z axis with the group velocity v_g.

IV. STORING AND RELEASING THE LIGHT

A. General

The probe beam $\mathcal{E}^{(s)}(t_i^{(s)})$ at time moment $t_i^{(s)}$ enters an atomic medium which is illuminated by two control beams characterized by the Rabi frequencies $\Omega_{c2}^{(s)}$ and $\Omega_{c3}^{(s)}$.

At the boundary, the probe beam is converted into a polariton propagating slowly in the medium with the velocity $v_g \ll c$. Here the index (s) refers to the stage of storing the light. Since the atomic population is created exclusively by the incident light field, the atomic dark-state Φ_D is not populated and, according to Eq. (18), the bright-state is

$$\Phi_B^{(s)}(t_i^{(s)}) = -g\frac{\Phi_1(t_i^{(s)})}{\Omega_c(t_i^{(s)})}\mathcal{E}^{(s)}(t_i^{(s)}). \tag{40}$$

Equations (16) and (17) give the atomic fields

$$\Phi_2^{(s)} = \xi_{c2}^{(s)*}\Phi_B^{(s)}, \quad \Phi_3^{(s)} = \xi_{c3}^{(s)*}\Phi_B^{(s)}. \tag{41}$$

To store the slow light, at the final time $t_f^{(s)}$ of the storing stage, both control fields are switched off in such a way that the ratios $\xi_{c2}^{(s)}$ and $\xi_{c3}^{(s)}$ remain constant whereas $\Omega_c^{(s)} \longrightarrow 0$. To restore the slow light propagation, the control fields are switched on again with relative Rabi frequencies $\xi_{c2}^{(r)}$ and $\xi_{c3}^{(r)}$. The latter can differ from the original ones $\xi_{c2}^{(s)}$ and $\xi_{c3}^{(s)}$, so the dark-state Φ_D can now be populated. Shortly after the beginning of the release of light (at $t = t_i^{(r)}$) the generated electric field reaches a steady state value, as described in the Subsection IIIC. Equation (33) yields the restored probe field

$$\mathcal{E}^{(r)}(t_i^{(r)}) = -\frac{\Omega_c^{(r)}(t_i^{(r)})}{g\Phi_1(t_i^{(r)})}(\xi_{c2}^{(r)}\xi_{c2}^{(s)*} + \xi_{c3}^{(r)}\xi_{c3}^{(s)*})\Phi_B^{(s)}(t_f^{(s)}), \tag{42}$$

where Eqs. (13), (22), and (41) were used to relate the bright state $\Phi_B^{(r)}(t_i^{(r)})$ of the restoring stage to the stored one $\Phi_B^{(s)}(t_f^{(s)})$.

B. Specific Cases

1. Restored control beams are proportional to the original ones. Supposed first that the Rabi frequencies of the restored control beams are proportional to the original ones: $\Omega_{c2}^{(r)} = a\Omega_{c2}^{(s)}$ and $\Omega_{c3}^{(r)} = a\Omega_{c3}^{(s)}$. For slow light this implies that

$$\xi_{c2}^{(r)} = \xi_{c2}^{(s)}, \quad \xi_{c3}^{(r)} = \xi_{c3}^{(s)}. \tag{43}$$

Since $\xi_{c2}\xi_{c2}^* + \xi_{c3}\xi_{c3}^* = 1$, Eq. (42) yields the following result for the regenerated electric field:

$$\mathcal{E}^{(r)}(t_i^{(r)}) = -\frac{\Omega_c^{(r)}(t_i^{(r)})}{g\Phi_1(t_i^{(r)})}\Phi_B^{(s)}(t_f^{(s)}). \tag{44}$$

The above relationship represents the initial condition for the subsequent propagation of the probe beam $\mathcal{E}$ governed by the equation of motion (37). The regenerated electric field is seen to acquire the phase from the bright polariton at its storage stage and the amplitude of $\mathcal{E}$ is modulated according to $\Omega_c^{(r)}$ at the release stage.

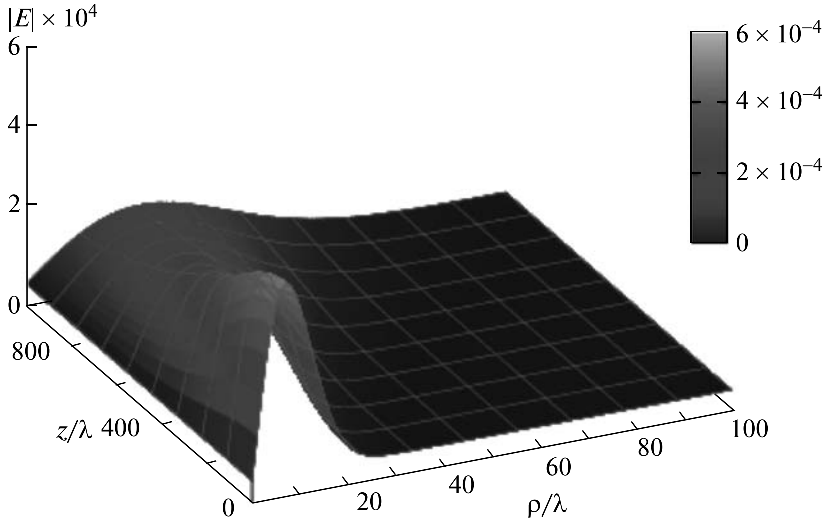

Fig. 2. Propagation of the regenerated electric field into the free space in the case of Λ system storing and tripod system retrieval.

2. System for the storage and tripod system for the retrieval. Suppose now that initially we have a Λ system with only a single control field: $\xi_{c3}^{(s)} = 0$ and hence $\left|\xi_{c2}^{(s)}\right| = 1$. On the other hand, a tripod system is used in the retrieval stage where generally both $\xi_{c2}^{(r)}$ and $\xi_{c3}^{(r)}$ are non-zero. In that case Eq. (42) provides the following result for the regenerated electric field:

$$\mathcal{E}^{(r)}(t_i^{(r)}) = -\frac{\Omega_{c2}^{(r)}(t_i^{(r)})}{g\Phi_1(t_i^{(r)})}\xi_{c2}^{(s)*}\Phi_B^{(s)}(t_f^{(s)}). \qquad (45)$$

The Eq. (45) represents the initial condition for the subsequent propagation of the probe beam $\mathcal{E}$ governed by the equation of motion (37) in the medium.

If the second control beam has an optical vortex at the restoring stage $\Omega_{c2}^{(r)} \sim e^{il\varphi}$, the regenerated electric field $\mathcal{E}^{(r)} \sim e^{il\varphi}$ acquires the same phase, as one can see from Eq. (45).

This means the restoring control beam transfers its optical vortex to the regenerated electric field $\mathcal{E}^{(r)}$. In the Λ scheme it is not allowed to have an optical vortex for the control beam due to adiabaticity violation at the center of the vortex. Using a tripod scheme for the regeneration lifts up this restriction. The probe beam may itself carry a vortex at the beginning of the storage [20]. Subsequently the vortex is stored onto the atomic bright state $\Phi_B^{(s)}$ and then transferred back to the probe beam after the control beam $\Omega_{c2}^{(r)}$ is turned on. In that case the phase of the restored vortex in the probe beam is defined by the product $\Omega_{c2}^{(r)}\Phi_B^{(s)}$ in which both $\Omega_{c2}^{(r)}$ and $\Phi_B^{(s)}$ may carry vortices. If these vortices have opposite winding numbers, they cancel each other leading to zero vorticity in the regenerated probe beam.

3. Tripod system for the storage and Λ system for and the retrieval. Suppose now that initially we have a tripod system for the storage where generally both $\xi_{c2}^{(s)}$ and $\xi_{c3}^{(s)}$ are non-zero. On the other hand, a Λ system is used in the retrieval stage with only one control field, i.e., $\xi_{c3}^{(r)} = 0$ and hence $\left|\xi_{c2}^{(r)}\right| = 1$. In that case Eq. (42) leads to the same Eq. (45) for the regenerated electric field. Yet now it is the storing control beam that has an optical vortex $\Omega_{c2}^{(s)} \sim e^{il\varphi}$. Subsequently the vortex is transferred to the regenerated probe field in the phase conjugated form: $\mathcal{E}^{(r)} \sim e^{-il\varphi}$.

V. PROPAGATION OF RESTORED FIELD

After the probe beam leaves the cold atomic cloud, it further propagates in free space according to Eq. (2) with $\mathbf{P} = 0$. The probe field is quasimonochromatic, so that its amplitude $\mathcal{E}(\mathbf{r}, t)$ varies very slowly in time during an optical cycle. In the time-independent case, we get the propagation equation

$$i\frac{\partial}{\partial z}\mathcal{E} = -\frac{1}{2k}\nabla_\perp^2\mathcal{E}. \qquad (46)$$

Since a typical length of the atomic cloud is not much larger than the length of the laser pulse, we will assume that the stored and restored electric fields do not change significantly during their propagation inside of the atomic cloud. Furthermore we will assume that the control beams are abruptly switched off during the storing stage and then switched on in the same way

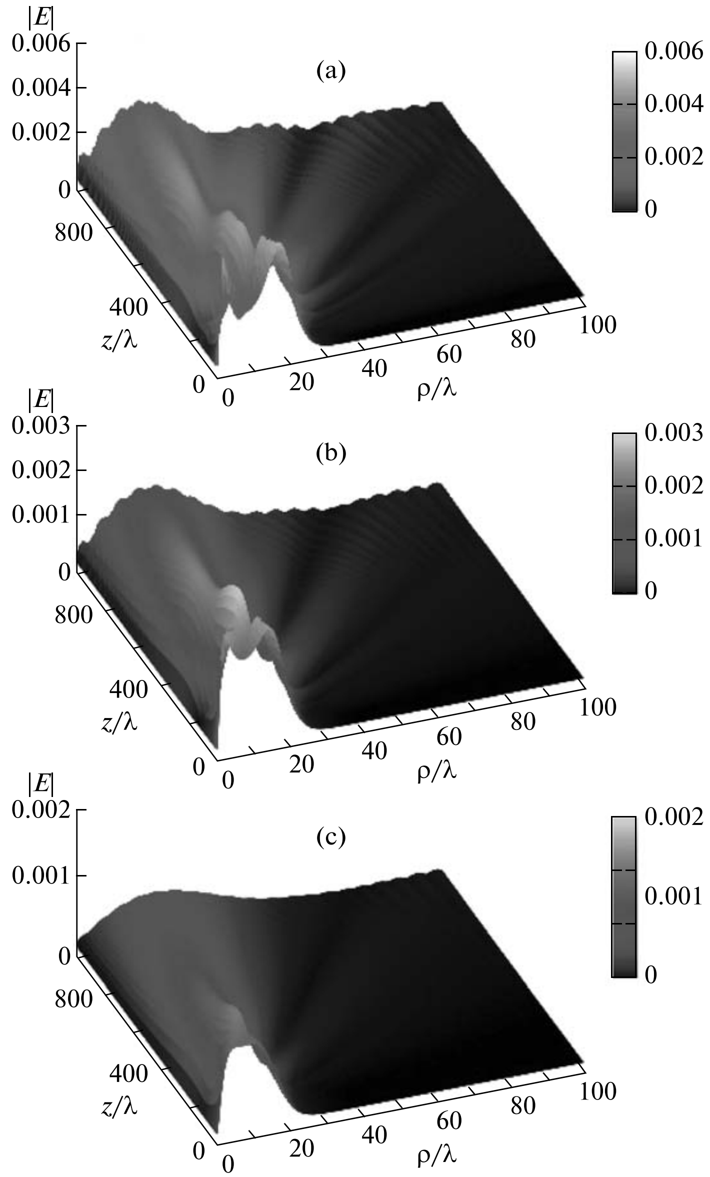

Fig. 3. Propagation of the regenerated electric field into free space in the case of tripod system storing and Λ system retrieval. Parameter *b* appearing in Eq. (51) is set to be equal to 1, 2 and 4, respectively, for the figures (a), (b), and (c).

when they are restored. Using Eq. (18) and Eq. (42), the restored field may be written as

$$\mathscr{E}^{(r)} = \frac{\Omega_c^{(r)}}{\Omega_c^{(s)}}(\xi_{c2}^{(r)}\xi_{c2}^{(s)*} + \xi_{c3}^{(r)}\xi_{c3}^{(s)*})\mathscr{E}^{(s)}.$$

In the following we will consider the case where the stored electric field is uniform, $\mathscr{E}^{(s)} = \text{const}$.

In the first case, the Λ system is used for the storage and the tripod system for the retrieval, so that $\xi_{c3}^{(s)} = 0$. Assuming that the control beams which do not carry vortices are uniform, the restored electric field (45) is proportional to the restoring control beam $\Omega_{c2}^{(r)}$ with some constant *A*:

$$\mathscr{E}^{(r)} = A\Omega_{c2}^{(r)}. \tag{47}$$

In the second case, the tripod system is used for the storage and Λ system for the retrieval ($\xi_{c3}^{(r)} = 0$). The expression for the restored electric field is then more complicated:

$$\mathscr{E}^{(r)} = A\frac{\Omega_{c2}^{(s)*}}{\left|\Omega_{c2}^{(s)}\right|^2 + \left|\Omega_{c3}^{(s)}\right|^2}. \tag{48}$$

Let us choose the vortex control field to be the first order Laguerre–Gaussian beam with $l = 1$:

$$\Omega_{c2} = a\rho e^{i\varphi}e^{-\rho^2/\sigma^2}. \quad (49)$$

We look for a solution of the regenerated electric field which leaves the atomic cloud of a form $\mathcal{E} = f(\rho, z)e^{il\varphi}$ with $l = \pm1$, where the positive (negative) vorticity applies to the first (second) case. In the dimensionless variables $\tilde{\rho} = \rho/\lambda$ and $\tilde{z} = z/\lambda$ the initial conditions for the equation of motion Eq. (46) reads

$$f(\tilde{\rho}, 0) = \tilde{\rho}e^{-\tilde{\rho}^2/\tilde{\sigma}^2} \quad (50)$$

for the first case and

$$f(\tilde{\rho}, 0) = \frac{\tilde{\rho}e^{-\tilde{\rho}^2/\tilde{\sigma}^2}}{\tilde{\rho}^2 e^{-2\tilde{\rho}^2/\tilde{\sigma}^2} + b^2} \quad (51)$$

for the second one (with b being the ratio $b = |\Omega_{c3}^{(s)}|/a$). With these two conditions, the Eq. (46) is solved numerically. The results for the first case is shown in Fig. 2. At the stage of release of light the electric field profile is defined by Eq. (50). The subsequent propagation of the field qualitatively preserves the profile distribution and is accompanied by the diffraction spreading.

The second case is illustrated in the next three figures (Figs. 3a–3c). In this case, the electric field profile is defined according to the Eq. (51) at the stage of the release of light. The initial profile and its subsequent development is determined by the parameter b which equals to 1 in Fig. 3a. Here the profile is distorted, the shape obviously being more complicated than that in the first case (Fig. 2). When b increases ($b = 2$ in Fig. 3b and $b = 4$ in Fig. 3c), the profile converges to a smooth shape similar to that shown in the previous Fig. 2. Extrapolation of the results suggests that we can increase the intensity of the second pump beam until the desired profile of the outgoing probe is achieved.

SUMMARY

In this article, we analyze the interaction of light and the cloud of cold atoms. Atoms are described by the three ground states and the excited one, which from a so-called tripod configuration. The application of an additional third laser ensures the adiabaticity of the atom-light coupling processes for the vortex beams. Using the adiabatic approximation we derive the equation of motion for the probe beam and analyze it in the case where one of the control beams has an optical vortex. Two cases of storing the vortex onto the atomic cloud are analyzed in more details. The first case involves the Λ system for the storage and tripod system for the retrieval. In such a situation, the phase of vortex is transferred from the restoring control beam to the regenerated probe beam. In the second case, we use the tripod system storing and the Λ system retrieval. The vortex phase is then transferred from the storing control beam to the regenerated probe in the phase conjugated form, so the probe beam acquires an opposite vorticity. Our numerical analysis for the restored fields shows that in the first case scenario the profile of the regenerated probe field is qualitatively preserved. In the second case, the additional storing control beam has to be strong enough for the spatial profile of the probe field to be preserved.

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