General expression for the quantum Zeno and anti-Zeno effects

J. Ruseckas* and B. Kaulakys
Institute of Theoretical Physics and Astronomy, Vilnius University, A. Goštauto 12, 2600 Vilnius, Lithuania
(Received 4 September 2003; published 12 March 2004)

In this paper we investigate the quantum Zeno and anti-Zeno effects without using any particular model of the measurement. Making a few assumptions about the measurement process we derive an expression for the jump probability during the measurement. From this expression the equation, obtained by Kofman and Kurizki [Nature (London) **405**, 546 (2000)] can be derived as a special case.

DOI: 10.1103/PhysRevA.69.032104 PACS number(s): 03.65.Xp, 03.65.Ta, 03.65.Yz, 42.50.Lc

I. INTRODUCTION

The description of the measurement process has been a problem since early development of quantum mechanics [1]. During recent years the measurement problem attracted much attention due to the advancement in experimental techniques. Nevertheless, the full understanding of quantum-mechanical measurements has not been achieved as yet. Typically, the measurement in quantum mechanics is described by von Neumann's state reduction (or projection) postulate [1]. However, this postulate refers only to an ideal measurement, which is instantaneous and arbitrarily accurate. Real measurements are represented by the projection postulate only roughly.

The so-called "quantum Zeno effect" is directly related to the measurement problem. In quantum mechanics the short-time behavior of nondecay probability of an unstable particle is not exponential but quadratic [2]. The deviation from the exponential decay has been observed by Wilkinson *et al.* [3]. Using the behavior of nondecay probability Misra and Sudarshan [4] in 1977 showed that the frequent observations can slow down the decay. An unstable particle would never decay when continuously observed. Misra and Sudarshan have called this effect the quantum Zeno paradox or quantum Zeno effect. The very first analysis does not take into account the actual mechanism of the measurement process involved, but it is based on an alternating sequence of unitary evolution and a collapse of the wave function. The quantum Zeno effect has been experimentally proved [5] in a repeatedly measured two-level system undergoing Rabi oscillations. The outcome of this experiment has also been explained without the collapse hypothesis [6–8].

Later it was realized that the repeated measurements could not only slow down the quantum dynamics but the quantum process may be accelerated by frequent measurements, as well [9–15]. This effect was called a quantum anti-Zeno effect. Quantum Zeno and anti-Zeno effect were experimentally observed in an atomic tunneling process [16].

Simple interpretation of quantum Zeno and anti-Zeno effects was given in Ref. [11]. Using projection postulate the universal formula describing both quantum Zeno and anti-Zeno effects was obtained. According to Ref. [11], the decay rate is determined by the convolution of two functions: the measurement-induced spectral broadening and the spectrum of the reservoir to which the decaying state is coupled.

In this paper we analyze the quantum Zeno and anti-Zeno effects without using any particular measurement model and making only few assumptions. We obtain a more general expression for the jump probability during the measurement. Expression derived in Ref. [11] is a special case of our formula.

The work is organized as follows. In Sec. II we present the description of the measurement. A simple case is considered in Sec. III. In Sec. IV we derived a general formula for the probability of the jump into another level during the measurement. The pulsed measurements when there is a period of the measurement-free evolution between the measurements is analyzed in Sec. V. Particular case of the expression, obtained in Sec. IV, is investigated in Sec. VI. Section VII summarizes our findings.

II. DESCRIPTION OF THE MEASUREMENT

We consider a system that consists of two parts. The first part of the system has the discrete energy spectrum. The Hamiltonian of this part is $\hat{H}_0$. The other part of the system is represented by Hamiltonian $\hat{H}_1$. Hamiltonian $\hat{H}_1$ commutes with $\hat{H}_0$. In a particular case the second part can be absent and $\hat{H}_1$ can be zero. The operator $\hat{V}(t)$ causes the jumps between different energy levels of $\hat{H}_0$. Therefore, the full Hamiltonian of the system is of the form $\hat{H}_S=\hat{H}_0+\hat{H}_1+\hat{V}(t)$. The example of such a system is an atom with the Hamiltonian $\hat{H}_0$ interacting with the electromagnetic field, represented by $\hat{H}_1$, while the interaction between the atom and the field is $\hat{V}(t)$.

We will measure in which eigenstate of the Hamiltonian $\hat{H}_0$ the system is. The measurement is performed by coupling the system with the detector. The full Hamiltonian of the system and the detector equals

$$\hat{H}=\hat{H}_S+\hat{H}_D+\hat{H}_I, \tag{1}$$

where $\hat{H}_D$ is the Hamiltonian of the detector and $\hat{H}_I$ represents the interaction between the detector and the measured system, described by the Hamiltonian $\hat{H}_0$. We can choose the

*Electronic address: ruseckas@itpa.lt

basis $|n\alpha\rangle = |n\rangle \otimes |\alpha\rangle$ common for the operators $\hat{H}_0$ and $\hat{H}_1$,

$$\hat{H}_0|n\rangle = E_n|n\rangle, \tag{2}$$

$$\hat{H}_1|\alpha\rangle = E_\alpha|\alpha\rangle, \tag{3}$$

where n numbers the eigenvalues of the Hamiltonian $\hat{H}_0$ and α represents the remaining quantum numbers.

The initial density matrix of the system is $\hat{\rho}_S(0)$. The initial density matrix of the detector is $\hat{\rho}_D(0)$. Before the measurement the measured system and the detector are uncorrelated, therefore, the full density matrix of the measured system and the detector is $\hat{\rho}(0) = \hat{\rho}_S(0) \otimes \hat{\rho}_D(0)$. The duration of the measurement is τ.

When the interaction of the detector with the environment is taken into account, the evolution of the measured system and the detector cannot be described by a unitary operator. More general description of the evolution, allowing to include the interaction with the environment, can be given using the superoperators. Therefore, we will assume that the evolution of the measured system and the detector is given by the superoperator $\mathcal{S}(t)$. The explicit form of the superoperator $\mathcal{S}(t)$ can be obtained from a concrete model of the measurement.

Due to the finite duration of the measurement it is impossible to realize the infinitely frequent measurements. The highest frequency of the measurements is achieved when the measurements are performed one after another without the period of the measurement-free evolution between two successive measurements. Therefore, we model a continuous measurement by the subsequent measurements of the finite duration and finite accuracy. After N measurements the full density matrix of the measured system and the detector is

$$\hat{\rho}(N\tau) = \mathcal{S}(\tau)^N \hat{\rho}(0). \tag{4}$$

We assume that the density matrix of the detector, $\hat{\rho}_D(0)$, is the same before each measurement. Such an assumption is valid when the initial condition for the detector, modified by the measurement, is restored at the beginning of each measurement or each measurement is performed with a new detector. For example, the detector can be an atom which is excited during the measurement. After the interaction of the atom with the measured system is interrupted, the atom returns to the ground state due to spontaneous emission, and the result of the measurement is encoded in the emitted photon. Thus the initial state of the detector is restored.

III. MEASUREMENT OF THE UNPERTURBED SYSTEM

In this section we investigate the measurement of the unperturbed system, i.e., the case when $V(t) = 0$.

We assume that the measurement of the unperturbed system is a quantum non-demolition measurement [17–20]. The measurement of the unperturbed system does not change the state of the measured system when initially the system is in an eigenstate of the Hamiltonian $\hat{H}_0$. After such an assumption, the most general form of the action of the superoperator $\mathcal{S}(\tau)$ can be written as

$$\begin{aligned}\mathcal{S}(\tau)[|n\alpha\rangle\langle m\alpha'| \otimes \hat{\rho}_D(0)] \\ = |n\alpha\rangle\langle m\alpha'| e^{i\omega_{m\alpha',n\alpha}\tau} \otimes \mathcal{S}_{n\alpha,m\alpha'}(\tau)\hat{\rho}_D(0),\end{aligned} \tag{5}$$

where

$$\omega_{m\alpha',n\alpha} = \frac{1}{\hbar}(E_m + E_{\alpha'} - E_n - E_\alpha) \tag{6}$$

and the superoperator $\mathcal{S}_{n\alpha,m\alpha'}(\tau)$ acts only on the density matrix of the detector. The full density matrix of the detector and the measured system after the measurement is

$$\begin{aligned}\hat{\rho}(\tau) = \mathcal{S}(\tau)\hat{\rho}(0) = \sum_{n\alpha,m\alpha'} |n\alpha\rangle(\rho_S)_{n\alpha,m\alpha'} e^{i\omega_{m\alpha',n\alpha}\tau}\langle m\alpha'| \\ \otimes \mathcal{S}_{n\alpha,m\alpha'}(\tau)\hat{\rho}_D(0).\end{aligned} \tag{7}$$

From Eq. (7) it follows that the nondiagonal matrix elements of the density matrix of the system after the measurement $(\rho_S)_{n\alpha,m\alpha'}(\tau)$ are multiplied by the quantity

$$F_{n\alpha,m\alpha'}(\tau) \equiv \mathrm{Tr}\{\mathcal{S}_{n\alpha,m\alpha'}(\tau)\hat{\rho}_D(0)\}. \tag{8}$$

Since after the measurement the nondiagonal matrix elements of the density matrix of the measured system should become small (they must vanish in the case of an ideal measurement), $F_{n\alpha,m\alpha'}(\tau)$ must be also small when $n \neq m$.

IV. MEASUREMENT OF THE PERTURBED SYSTEM

The operator $\hat{V}(t)$ represents the perturbation of the unperturbed Hamiltonian $\hat{H}_0 + \hat{H}_1$. We will take into account the influence of the operator $\hat{V}(t)$ by the perturbation method, assuming that the strength of the interaction between the system and detector is large and the duration of the measurement τ is short. Similar method was used in Ref. [21].

We assume that the Markovian approximation is valid, i.e., the evolution of the measured system and the detector depends only on their state at the present time. Then the superoperator $\mathcal{S}$, describing the evolution of the measured system and the detector, obeys the equation

$$\frac{\partial}{\partial t}\mathcal{S} = \mathcal{L}(t)\mathcal{S}, \tag{9}$$

where $\mathcal{L}$ is the Liouvillian. There is a small perturbation of the measured system, given by the operator $\hat{V}$. We can write $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_V$, where $\mathcal{L}_V$ is a small perturbation. We expand the superoperator $\mathcal{S}$ into powers of V,

$$\mathcal{S} = \mathcal{S}^{(0)} + \mathcal{S}^{(1)} + \mathcal{S}^{(2)} + \cdots. \tag{10}$$

Then from Eq. (9) it follows

$$\frac{\partial}{\partial t}\mathcal{S}^{(0)} = \mathcal{L}_0(t)\mathcal{S}^{(0)}, \tag{11}$$

$$\frac{\partial}{\partial t}\mathcal{S}^{(i)} = \mathcal{L}_0(t)\mathcal{S}^{(i)} + \mathcal{L}_V(t)\mathcal{S}^{(i-1)}. \quad (12)$$

We will denote as $\mathcal{S}^{(0)}(t,t_0)$ the solution of Eq. (11) with the initial condition $\mathcal{S}^{(0)}(t=t_0,t_0)=1$. The formal solutions of Eqs. (11) and (12) are

$$\mathcal{S}^{(0)}(t,t_0) = T\exp\left(\int_{t_0}^{t}\mathcal{L}_0(t')dt'\right) \quad (13)$$

and

$$\mathcal{S}^{(i)}(t,0) = \int_0^t dt_1\mathcal{S}^{(0)}(t,t_1)\mathcal{L}_V(t_1)\mathcal{S}^{(i-1)}(t_1,0). \quad (14)$$

Here T represents the time ordering. In the second-order approximation we have

$$\begin{aligned}\mathcal{S}(t,0) = \mathcal{S}^{(0)}(t,0) &+ \int_0^t dt_1\mathcal{S}^{(0)}(t,t_1)\mathcal{L}_V(t_1)\mathcal{S}^{(0)}(t_1,0)\\ &+ \int_0^t dt_1\int_0^{t_1} dt_2\mathcal{S}^{(0)}(t,t_1)\mathcal{L}_V(t_1)\mathcal{S}^{(0)}(t_1,t_2)\\ &\times\mathcal{L}_V(t_2)\mathcal{S}^{(0)}(t_2,0). \end{aligned} \quad (15)$$

Using Eq. (10), the full density matrix of the measured system and the detector can be represented as

$$\hat{\rho}(t) = \hat{\rho}^{(0)}(t) + \hat{\rho}^{(1)}(t) + \hat{\rho}^{(2)}(t) + \cdots, \quad (16)$$

where

$$\hat{\rho}^{(i)}(t) = \mathcal{S}^{(i)}(t,0)\hat{\rho}(0). \quad (17)$$

Let the initial density matrix of the system and detector be

$$\hat{\rho}(0) = |i\alpha\rangle\langle i\alpha| \otimes \hat{\rho}_D(0). \quad (18)$$

The probability of the jump from the level $|i\alpha\rangle$ into the level $|f\alpha'\rangle$ during the measurement is

$$W(i\alpha \to f\alpha') = \mathrm{Tr}\{|f\alpha'\rangle\langle f\alpha'|\hat{\rho}(\tau)\}. \quad (19)$$

Using Eq. (5) we can write

$$\begin{aligned}&\mathcal{S}^{(0)}(t,t_0)[|n\alpha\rangle\langle m\alpha'| \otimes \hat{\rho}_D(0)]\\ &\quad = |n\alpha\rangle\langle m\alpha'|e^{i\omega_{m\alpha',n\alpha}t} \otimes \mathcal{S}^{(0)}_{n\alpha,m\alpha'}(t,t_0)\hat{\rho}_D(0). \end{aligned}\quad (20)$$

From Eq. (20) it follows that the superoperator $\mathcal{S}^{(0)}_{m\alpha,m\alpha}$ with the equal indices does not change the trace of the density matrix $\hat{\rho}_D$, since the trace of the full density matrix of the measured system and the detector must remain unchanged during the evolution.

When the system is perturbed by the operator $\hat{V}(t)$ then the superoperator $\mathcal{L}_V$ is defined by the equation

$$\mathcal{L}_V(t)\hat{\rho} = \frac{1}{i\hbar}[\hat{V}(t),\hat{\rho}]. \quad (21)$$

The first-order term is $\hat{\rho}^{(1)}(t)=\mathcal{S}^{(1)}(t,0)\hat{\rho}(0)$. Using Eqs. (14), (18), (20), and (21), this term can be written as

$$\begin{aligned}\hat{\rho}^{(1)}(t) = \sum_{p\alpha_1}\frac{1}{i\hbar}\int_0^t dt_2[&|p\alpha_1\rangle V_{p\alpha_1,i\alpha}(t_2)e^{i\omega_{i\alpha,p\alpha_1}(t-t_2)}\langle i\alpha|\\ &\otimes \mathcal{S}^{(0)}_{p\alpha_1,i\alpha}(t,t_2) - |i\alpha\rangle V_{i\alpha,p\alpha_1}(t_2)e^{i\omega_{p\alpha_1,i\alpha}(t-t_2)}\langle p\alpha_1|\\ &\otimes \mathcal{S}^{(0)}_{i\alpha,p\alpha_1}(t,t_2)]\mathcal{S}^{(0)}_{i\alpha,i\alpha}(t_2,0)\hat{\rho}_D(0). \end{aligned}\quad (22)$$

When $i\neq f$ then the first-order term does not contribute to the jump probability, since from Eqs. (19) and (22) it follows that the expression for this contribution contains the scalar product $\langle f\alpha'|i\alpha\rangle=0$.

For the second-order term $\hat{\rho}^{(2)}(t)=\mathcal{S}^{(2)}(t,0)\hat{\rho}(0)$, using Eqs. (14) and (20), we obtain the equality

$$\begin{aligned}\mathrm{Tr}\{|f\alpha'\rangle\langle f\alpha'|\hat{\rho}^{(2)}(t)\} = \frac{1}{i\hbar}\int_0^t dt_1\mathrm{Tr}\{&\langle f\alpha'|\hat{V}(t_1)\hat{\rho}^{(1)}(t_1)|f\alpha'\rangle\\ &- \langle f\alpha'|\hat{\rho}^{(1)}(t_1)\hat{V}(t_1)|f\alpha'\rangle\}. \end{aligned}\quad (23)$$

In Eq. (23) the superoperator $\mathcal{S}^{(0)}_{f\alpha',f\alpha'}$ is omitted, since it does not change the trace. Then from Eqs. (22) and (23) we obtain the jump probability

$$\begin{aligned}&W(i\alpha \to f\alpha')\\ &\quad = \frac{1}{\hbar^2}\int_0^\tau dt_1\int_0^{t_1} dt_2\mathrm{Tr}\{[V_{f\alpha',i\alpha}(t_1)V_{i\alpha,f\alpha'}(t_2)\\ &\qquad \times S^{(0)}_{i\alpha,f\alpha'}(t_1,t_2)e^{i\omega_{f\alpha',i\alpha}(t_1-t_2)} + V_{f\alpha',i\alpha}(t_2)V_{i\alpha,f\alpha'}(t_1)\\ &\qquad \times \mathcal{S}^{(0)}_{f\alpha',i\alpha}(t_1,t_2)e^{i\omega_{i\alpha,f\alpha'}(t_1-t_2)}]\mathcal{S}^{(0)}_{i\alpha,i\alpha}(t_2,0)\hat{\rho}_D(0)\}. \end{aligned}\quad (24)$$

Equation (24) allows us to calculate the jump probability during the measurement when the evolution of the measured unperturbed system is known. The explicit form of the superoperator $\mathcal{S}^{(0)}_{n\alpha,m\alpha'}$ can be obtained from a concrete model of the measurement. The main assumptions, used in the derivation of Eq. (24), are Eqs. (5) and (9), i.e., the assumptions that the quantum measurement of the unperturbed system is nondemolition measurement and that the Markovian approximation is valid. Thus, Eq. (24) is quite general.

The probability that the measured system remains in the initial state $|i\alpha\rangle$ is

$$W(i\alpha) = 1 - \sum_{f,\alpha'} W(i\alpha \to f\alpha'). \quad (25)$$

After N measurements the probability that the measured system remains in the initial state equals

$$W(i\alpha)^N \approx \exp(-RN\tau), \quad (26)$$

where R is the jump rate,

$$R = \sum_{f,\alpha'}\frac{1}{\tau}W(i\alpha \to f\alpha'). \quad (27)$$

V. FREE EVOLUTION AND MEASUREMENTS

In practice, it is impossible to perform the measurements one after another without the period of the measurement-free evolution between two successive measurements. Such intervals of the measurement-free evolution were also present in the experiments demonstrating the quantum Zeno effect [5,16,22]. Therefore, it is important to consider such measurements. This problem for the definite model was investigated in Ref. [23].

We have the repeated measurements separated by the free evolution of the measured system. For the purpose of the description of such measurements we can use Eq. (24), obtained in Sec. IV. The duration of the free evolution is τ_F, and the duration of the free evolution and the measurement together is τ. The superoperator of the free evolution without the perturbation $\hat{V}$ is $\mathcal{S}_F^{(0)}(t)$, and the superoperator of the measurement is $\mathcal{S}_M^{(0)}(t,t_0)$. We will assume that during the measurement the superoperator $\mathcal{L}_0$ does not depend on time t. Then the superoperator $\mathcal{S}_M^{(0)}(t,t_0)$ depends only on the time difference $t-t_0$. Therefore, we will write $S_M^{(0)}(t-t_0)$ instead of $\mathcal{S}_M^{(0)}(t,t_0)$. When the free evolution comes first and then the measurement is performed, the full superoperator equals

$$\mathcal{S}^{(0)}_{n\alpha,m\alpha'}(t,t_1)=\begin{cases}\mathcal{S}^{(0)}_{M\ n\alpha,m\alpha'}(t-t_1), & \tau>t_1>\tau_F \text{ and } \tau>t>t_1\\ \mathcal{S}^{(0)}_F(t-t_1), & \tau_F>t_1>0 \text{ and } \tau_F>t>t_1\\ \mathcal{S}^{(0)}_{M\ n\alpha,m\alpha'}(t-\tau_F)\mathcal{S}^{(0)}_F(\tau_F-t_1), & \tau_F>t_1>0 \text{ and } \tau>t>\tau_F.\end{cases} \tag{28}$$

Equation (28) can be written as

$$\begin{aligned}S^{(0)}_{n\alpha,m\alpha'}(t,t_1)&=\mathcal{S}^{(0)}_{M\ n\alpha,m\alpha'}(t-t_1)\Theta(t_1-\tau_F)\\&\quad+\mathcal{S}^{(0)}_F(t-t_1)\Theta(\tau_F-t)+\mathcal{S}^{(0)}_{M\ n\alpha,m\alpha'}(t-\tau_F)\\&\quad\times\mathcal{S}^{(0)}_F(\tau_F-t_1)\Theta(t-\tau_F)\Theta(\tau_F-t_1),\end{aligned} \tag{29}$$

where Θ is Heaviside unit step function. From Eqs. (24) and (29) it follows that the jump probability consists of three terms

$$\begin{aligned}W(i\alpha\to f\alpha')&=W_M(i\alpha\to f\alpha')+W_F(i\alpha\to f\alpha')\\&\quad+W_I(i\alpha\to f\alpha'),\end{aligned} \tag{30}$$

where the jump probability during the free evolution is

$$\begin{aligned}&W_F(i\alpha\to f\alpha')\\&\quad=\frac{1}{\hbar^2}\int_0^{\tau_F}dt_1\int_0^{\tau_F}dt_2V_{f\alpha',i\alpha}(t_1)V_{i\alpha,f\alpha'}(t_2)e^{i\omega_{f\alpha',i\alpha}(t_1-t_2)},\end{aligned} \tag{31}$$

the jump probability during the measurement

$$\begin{aligned}&W_M(i\alpha\to f\alpha')\\&\quad=\frac{1}{\hbar^2}\int_{\tau_F}^{\tau}dt_1\int_{\tau_F}^{t_1}dt_2\mathrm{Tr}\{[V_{f\alpha',i\alpha}(t_1)V_{i\alpha,f\alpha'}(t_2)\\&\qquad\times\mathcal{S}^{(0)}_{M\ i\alpha,f\alpha'}(t_1-t_2)e^{i\omega_{f\alpha',i\alpha}(t_1-t_2)}\\&\qquad+V_{f\alpha',i\alpha}(t_2)V_{i\alpha,f\alpha'}(t_1)\mathcal{S}^{(0)}_{M\ f\alpha',i\alpha}(t_1-t_2)e^{i\omega_{i\alpha,f\alpha'}(t_1-t_2)}]\\&\qquad\times\mathcal{S}^{(0)}_{M\ i\alpha,i\alpha}(t_2-\tau_F)\mathcal{S}^{(0)}_F(\tau_F)\hat{\rho}_D(0)\},\end{aligned} \tag{32}$$

and the interference term is

$$\begin{aligned}&W_I(i\alpha\to f\alpha')\\&\quad=\frac{1}{\hbar^2}\int_{\tau_F}^{\tau}dt_1\int_0^{\tau_F}dt_2\mathrm{Tr}\{[V_{f\alpha',i\alpha}(t_1)V_{i\alpha,f\alpha'}(t_2)\\&\qquad\times\mathcal{S}^{(0)}_{M\ i\alpha,f\alpha'}(t_1-\tau_F)e^{i\omega_{f\alpha',i\alpha}(t_1-t_2)}+V_{f\alpha',i\alpha}(t_2)V_{i\alpha,f\alpha'}(t_1)\\&\qquad\times\mathcal{S}^{(0)}_{M\ f\alpha',i\alpha}(t_1-\tau_F)e^{i\omega_{i\alpha,f\alpha'}(t_1-t_2)}]\mathcal{S}^{(0)}_F(\tau_F)\hat{\rho}_D(0)\}.\end{aligned} \tag{33}$$

If we assume that the free evolution does not change the density matrix of the detector and the perturbation $\hat{V}$ does not depend on time, we have the jump probability during the measurement-free evolution

$$W_F(i\alpha\to f\alpha')=|V_{i\alpha,f\alpha'}|^2\frac{4\sin^2\left(\frac{1}{2}\omega_{f\alpha',i\alpha}\tau_F\right)}{\hbar^2\omega^2_{f\alpha',i\alpha}}, \tag{34}$$

the jump probability during the measurement

$$\begin{aligned}&W_M(i\alpha\to f\alpha')\\&\quad=\frac{1}{\hbar^2}|V_{i\alpha,f\alpha'}|^2\int_{\tau_F}^{\tau}dt_1\int_{\tau_F}^{t_1}dt_2\\&\qquad\times\mathrm{Tr}\{[\mathcal{S}^{(0)}_{M\ i\alpha,f\alpha'}(t_1-t_2)e^{i\omega_{f\alpha',i\alpha}(t_1-t_2)}\\&\qquad+\mathcal{S}^{(0)}_{M\ f\alpha',i\alpha}(t_1-t_2)e^{i\omega_{i\alpha,f\alpha'}(t_1-t_2)}]\\&\qquad\times\mathcal{S}^{(0)}_{M\ i\alpha,i\alpha}(t_2-\tau_F)\hat{\rho}_D(0)\},\end{aligned} \tag{35}$$

and the interference term

$$\begin{aligned}W_I(i\alpha\to f\alpha')&=|V_{i\alpha,f\alpha'}|^2\frac{2\sin\left(\frac{1}{2}\omega_{f\alpha',i\alpha}\tau_F\right)}{\hbar^2\omega_{f\alpha',i\alpha}}\\&\qquad\times\int_{\tau_F}^{\tau}dt_1\mathrm{Tr}\{[\mathcal{S}^{(0)}_{M\ i\alpha,f\alpha'}(t_1-\tau_F)\\&\qquad\times e^{i\omega_{f\alpha',i\alpha}\left(t_1-\frac{1}{2}\tau_F\right)}+\mathcal{S}^{(0)}_{M\ f\alpha',i\alpha}(t_1-\tau_F)\end{aligned}$$

$$\times e^{i\omega_{i\alpha,f\alpha'}\left(t_1-\frac{1}{2}\tau_F\right)}]\hat{\rho}_D(0)\}. \qquad (36)$$

VI. SIMPLIFICATION OF THE EXPRESSION FOR THE JUMP PROBABILITY

The expression for the jump probability during the measurement can be simplified if the operator $\hat{V}$ does not depend on time t. Then Eq. (24) can be written as

$$W(i\alpha \to f\alpha') = \frac{2}{\hbar^2}|V_{i\alpha,f\alpha'}|^2 \mathrm{Re}\int_0^\tau dt_1 \int_0^{t_1} dt_2 e^{i\omega_{f\alpha',i\alpha}(t_1-t_2)} \times \mathrm{Tr}\{\mathcal{S}^{(0)}_{i\alpha,f\alpha'}(t_1,t_2)\mathcal{S}^{(0)}_{i\alpha,i\alpha}(t_2,0)\hat{\rho}_D(0)\}. \qquad (37)$$

Introducing the function

$$G(\omega)_{f\alpha',i\alpha} = |V_{i\alpha,f\alpha'}|^2 \delta\left(\frac{1}{\hbar}(E_{\alpha'} - E_\alpha) - \omega\right), \qquad (38)$$

we can rewrite Eq. (37) in the form

$$W(i\alpha \to f\alpha') = \frac{2\pi\tau}{\hbar^2}\int_{-\infty}^{\infty} G(\omega)_{f\alpha',i\alpha} P(\omega)_{i\alpha,f\alpha'} d\omega, \qquad (39)$$

where

$$P(\omega)_{i\alpha,f\alpha'} = \frac{1}{\pi\tau}\mathrm{Re}\int_0^\tau dt_1 \int_0^{t_1} dt_2 e^{i(\omega-\omega_{if})(t_1-t_2)} \mathrm{Tr}\{\mathcal{S}^{(0)}_{i\alpha,f\alpha'} \times (t_1,t_2)\mathcal{S}^{(0)}_{i\alpha,i\alpha}(t_2,0)\hat{\rho}_D(0)\}. \qquad (40)$$

Equation (39) is similar to that obtained by Kofman and Kurizki in Ref. [11].

Further simplification can be achieved when the superoperator $\mathcal{L}_0$ does not depend on time t and the order of the superoperators in the expression

$$\mathrm{Tr}\{\mathcal{S}^{(0)}_{i\alpha,f\alpha'}(t_1,t_2)\mathcal{S}^{(0)}_{i\alpha,i\alpha}(t_2)\hat{\rho}_D(0)\}$$

can be changed. Under such assumptions we have

$$\begin{aligned}\mathrm{Tr}\{\mathcal{S}^{(0)}_{i\alpha,f\alpha'}(t_1,t_2)\mathcal{S}^{(0)}_{i\alpha,i\alpha}(t_2)\hat{\rho}_D(0)\} &= \mathrm{Tr}\{\mathcal{S}^{(0)}_{i\alpha,i\alpha}(t_2)\mathcal{S}^{(0)}_{i\alpha,f\alpha'}(t_1,t_2)\hat{\rho}_D(0)\} \\ &= F_{i\alpha,f\alpha'}(t_1 - t_2),\end{aligned} \qquad (41)$$

where $F_{i\alpha,f\alpha'}(t)$ is defined by Eq. (8). After changing the variables into $u=t_1-t_2$ and $v=t_1+t_2$ from Eq. (40) we obtain

$$P(\omega)_{i\alpha,f\alpha'} = \frac{1}{\pi}\mathrm{Re}\int_0^\tau \left(1 - \frac{u}{\tau}\right)F_{i\alpha,f\alpha'}(u)\exp[i(\omega - \omega_{if})u]du. \qquad (42)$$

Decaying system

We consider a decaying system with the Hamiltonian $\hat{H}_0$ that due to the interaction with the field decays from the level $|i\rangle$ into the level $|f\rangle$. The field initially is in the vacuum state $|\alpha=0\rangle$. Only the energy levels of the decaying system are measured and the detector does not interact with the field. Then $\mathcal{S}^{(0)}_{i\alpha,f\alpha'}$ and $P(\omega)_{i\alpha,f\alpha'}$ do not depend on α and α'. Using Eqs. (27) and (39) we obtain the decay rate of the measured system,

$$R = \sum_\alpha \frac{1}{\tau}W(i0 \to f\alpha) = \frac{2\pi}{\hbar^2}\int_{-\infty}^{\infty} G(\omega)_{f,i}P(\omega)_{i,f}d\omega, \qquad (43)$$

where

$$G(\omega)_{f,i} = \sum_\alpha G(\omega)_{f\alpha,i0}. \qquad (44)$$

The function $P(\omega)_{i,f}$ is related to the measurement-induced broadening of the spectral line [11,14,15]. For example, when instantaneous ideal measurements are performed at time intervals τ, we can take $F_{i\alpha,f\alpha'}(t)=\Theta(\tau-t)$, where $\Theta(t)$ is the unit step function. Then from Eq. (42) we get

$$P(\omega)_{i,f} = \frac{2\sin^2\left[\frac{1}{2}\tau(\omega - \omega_{if})\right]}{\pi\tau(\omega - \omega_{if})^2}.$$

We have that the width of the function $P(\omega)_{i,f}$ increases when the duration of the measurement τ decreases.

Equation (39) represents a universal result: the decay rate of the frequently measured decaying system is determined by the overlap of the reservoir coupling spectrum $G(\omega)_{f,i}$ and the measurement-modified level width $P(\omega)_{i,f}$.

Depending on the reservoir spectrum $G(\omega)_{f,i}$ and the frequency of the measurements $1/\tau$ the inhibition or acceleration of the decay can be obtained. If the frequency of measurements is small and, consequently, the measurement-induced broadening of the spectral line is much smaller than the width of the reservoir coupling spectrum, the decay rate equals the decay rate of the unmeasured system, given by the Fermi's golden rule. In the intermediate region, when the width of the spectral line is rather small compared with the distance between ω_{if} and the nearest maximum in the reservoir spectrum, the decay rate grows with increase of the frequency of the measurements. This results in the anti-Zeno effect.

If the width of the spectral line is much greater compared both with the width of the reservoir spectrum and the distance between ω_{if} and the centrum of the reservoir spectrum, the decay rate decreases when the frequency of measurements increases. This results in the quantum Zeno effect.

VII. CONCLUSIONS

We analyze the quantum Zeno and quantum anti-Zeno effects without using any particular model of the measurement. The general expression (24) for the jump probability during the measurement is derived. The main assumptions, used in the derivation of Eq. (24), are assumptions that the quantum measurement is nondemolition measurement [Eq. (5)] and the Markovian approximation for the quantum dynamics is valid [Eq. (9)]. We have shown that Eq. (24) is also suitable for the description of the pulsed measurements, when there are intervals of the measurement-free evolution between successive measurements [Eqs. (30)–(33)]. When the operator $\hat{V}$ inducing the jumps from one state to another does not depend on time Eq. (39), which is of the form obtained by Kofman and Kurizki [11], is derived as a special case.

[1] J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, 1955).
[2] L. A. Khalfin, Zh. Eksp. Teor. Fiz. **33**, 1371 (1958) [Sov. Phys. JETP **6**, 1503 (1958)].
[3] S. Wilkinson, C. Bharucha, M. Fischer, K. Madison, P. Morrow, Q. Niu, B. Sundaram, and M. Raizen, Nature (London) **387**, 575 (1997).
[4] B. Misra and E. C. G. Sudarshan, J. Math. Phys. **18**, 756 (1977).
[5] W. M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland, Phys. Rev. A **41**, 2295 (1990).
[6] T. Petrosky, S. Tasaki, and I. Prigogine, Phys. Lett. A **151**, 109 (1990).
[7] V. Frerichs and A. Schenzle, Phys. Rev. A **44**, 1962 (1991).
[8] S. Pascazio and M. Namiki, Phys. Rev. A **50**, 4582 (1994).
[9] A. G. Kofman and G. Kurizki, Phys. Rev. A **54**, R3750 (1996).
[10] B. Kaulakys and V. Gontis, Phys. Rev. A **56**, 1131 (1997).
[11] A. G. Kofman and G. Kurizki, Nature (London) **405**, 546 (2000).
[12] M. Lewenstein and K. Rzążewski, Phys. Rev. A **61**, 022105 (2000).
[13] P. Facchi, H. Nakazato, and S. Pascazio, Phys. Rev. Lett. **86**, 2699 (2001).
[14] J. Ruseckas and B. Kaulakys, Phys. Rev. A **63**, 062103 (2001).
[15] A. G. Kofman, G. Kurizki, and T. Opatrny, Phys. Rev. A **63**, 042108 (2001).
[16] M. C. Fischer, B. Gutiérrez- Medina, and M. G. Raizen, Phys. Rev. Lett. **87**, 040402 (2001).
[17] V. B. Braginsky, Y. I. Vorontsov, and K. S. Thorne, Science **209**, 547 (1980).
[18] C. M. Caves, K. S. Thorne, R. W. P. Drever, V. D. Sandberg, and M. Zimmermann, Rev. Mod. Phys. **52**, 341 (1980).
[19] W. G. Unruh, Phys. Rev. D **19**, 2888 (1979).
[20] V. B. Braginsky and F. Y. Khalili, Rev. Mod. Phys. **68**, 1 (1996).
[21] J. Ruseckas, Phys. Rev. A **66**, 012105 (2002).
[22] C. Balzer, R. Huesmann, W. Neuhauser, and P. E. Toschek, Opt. Commun. **180**, 115 (2000).
[23] J. Ruseckas, Phys. Lett. A **291**, 185 (2001).