Transfer of orbital angular momentum of light using two-component slow light

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We study the manipulation of slow light with an orbital angular momentum propagating in a cloud of cold atoms. Atoms are affected by four co-propagating control laser beams in a double tripod configuration of the atomic energy levels involved, allowing us to minimize the losses at the vortex core of the control beams. In such a situation the atomic medium is transparent for a pair of co-propagating probe fields, leading to the creation of two-component (spinor) slow light. We study the interaction between the probe fields when two control beams carry optical vortices of opposite helicity. As a result, a transfer of the optical vortex takes place from the control to the probe fields without switching off and on the control beams. This feature is missing in a single tripod scheme where the optical vortex can be transferred from the control to the probe field only during either the storage or retrieval of light.

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I. INTRODUCTION

Distinctive properties of slow [1–5], stored [6–18], and stationary [19–24] light have been extensively studied for more than a decade. The research has been motivated both by the fundamental interest in the slow and stationary light and also because of the potential applications including interalia the reversible quantum memories [6,9,10,13,25–29] and nonlinear optics at low intensities [30–34]. The slow light is formed due to the phenomenon known as the electromagnetically induced transparency (EIT) [25,26,35–37]. The EIT emerges in a medium resonantly driven by several laser fields and involves the destructive quantum interference between different resonant excitation pathways of atoms. As a result a weaker (probe) beam of light tuned to an atomic resonance can propagate slowly and is almost lossless when the medium is driven by one or several control beams of light with a higher intensity. Under suitable conditions reshaping of the dispersive properties of the medium by the control beams leads to a drastic reduction in the group velocity of the probe pulse. Group velocities as small as several of tens of meters per second have been reported [1–5]. Most of the work on the slow light dealt with a single probe beam and one or several control beams resonantly interacting with atomic media, with Λ configuration of the atom-light coupling being one of the most exploited [25,26,35–37].

Recently it was suggested to create a two-component slow light using a more complex double tripod setup [38,39] involving three ground atomic states coupled with two excited states. Such a setup supports a simultaneous propagation of two probe beams and leads to the formation of a two-component slow light. By properly choosing the control lasers one can generate a tunable coupling between the constituent probe fields.

The orbital angular momentum (OAM) [40–43] provides an additional possibility in manipulating the slow light. The optical OAM represents an extra degree of freedom which can be exploited in the quantum computation and quantum information storage [43]. Most of the previous studies on the vortex slow light considered situations where the incident probe beam carries an OAM [44–48], yet the control beam has no vortex. Application of a vortex control beam causes a potential problem, because its intensity goes to zero at the vortex core leading to the disappearance of the EIT accompanied with the absorption losses in this spatial region. To avoid such losses it was suggested [49] to employ an extra control laser beam without an optical vortex making a more complex tripod scheme of the atom-light coupling previously considered for the nonvortex beams of light [50–60]. The total intensity of the control lasers is then nonzero at the vortex core of the first control laser thus avoiding the losses. Using such a scheme a transfer of an optical vortex can be accomplished during switching off and on the control beams [49].

Here we show that the transfer of the vortex between the control and probe beams can be accomplished without switching off and on the control beams using a more complex double tripod scheme of the atom-light coupling. The scheme shown in Fig. 1 involves three atomic ground states coupled to two excited states via four control beams, two of them carrying optical vortices. If the incoming probe beam does not carry an optical vortex, the coupling with the control beams generates another component of the probe beam containing an OAM. We explore the efficiency of such a transfer of the optical vortex. We analyze the losses resulting from the exchange of the optical vortex between the control and probe beams, and provide conditions for the optical vortex of the control beam to be transferred efficiently to the second component of the probe beam.

The paper is organized as follows. In Sec. II we present the double tripod scheme and the equations for atomic operators and probe fields. In Sec. III we derive equations of propagation for the probe fields using an adiabatic approximation. We use those equations in Sec. IV for the description of the transfer of OAM from control to probe beams. Section V summarizes the findings.

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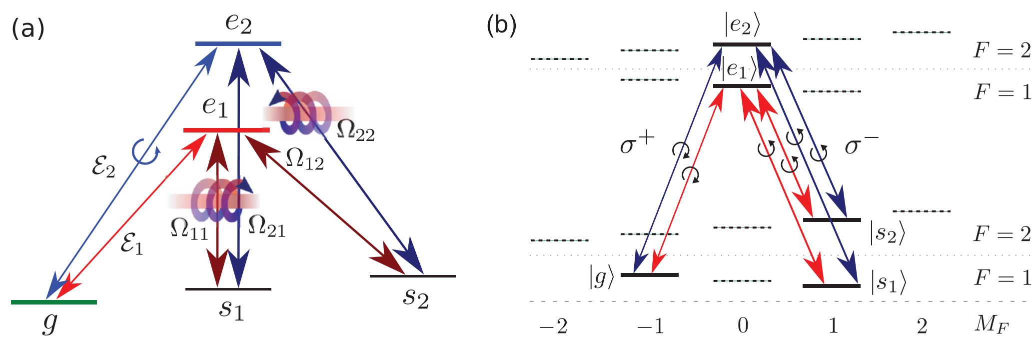

FIG. 1. (Color online) (a) Double tripod level scheme. (b) Possible experimental realization of the double tripod setup for atoms like rubidium [8] or sodium [7]. The scheme involves transitions between the magnetic states of two hyperfine levels with $F = 1$ and $F = 2$ for the ground- and excited-state manifolds. Both probe beams are circular σ^+ polarized and all four control beams are circular σ^- polarized.

II. FORMULATION

We shall consider the light-matter interaction in an ensemble of atoms using a double tripod coupling scheme shown in Fig. 1(a). The atoms are characterized by three hyperfine ground levels $|g\rangle$, $|s_1\rangle$, and $|s_2\rangle$ which have dipole-allowed optical transitions to the electronic excited-state levels $|e_1\rangle$ and $|e_2\rangle$. The atom-light coupling scheme involves two laser fields of low intensity (probe fields) and four fields of much higher intensity (control fields). The probe beams are described by the electric-field amplitudes $\mathcal{E}_1$ and $\mathcal{E}_2$ with the corresponding central frequencies ω_1 and ω_2. They drive atomic transitions $|g\rangle \rightarrow |e_1\rangle$ and $|g\rangle \rightarrow |e_2\rangle$, characterized by the dipole moments μ_1 and μ_2. Control fields having frequencies ω_{jq} couple the atomic transitions $|s_1\rangle \rightarrow |e_1\rangle$ and $|s_2\rangle \rightarrow |e_2\rangle$ with coupling strength being characterized by Rabi frequencies Ω_{jq}, $(j,q = 1,2)$. We assume the four-photon resonances with the probe beams for each pair of the control lasers, meaning that $\omega_1 - \omega_{1q} = \omega_2 - \omega_{2q}$. The presence of the control beams makes the medium transparent for the resonant probe beams in a narrow frequency range due to the electromagnetically induced transparency (EIT). To satisfy the EIT conditions probe fields should be quasimonochromatic requiring that the amplitudes $\mathcal{E}_j$ change a little during the optical cycle.

The double tripod scheme can be implemented using atoms like rubidium or sodium which contain two hyperfine ground levels with $F = 1$ and $F = 2$, as illustrated in Fig. 1(b). Such atoms have been used in the initial light-storage experiments involving a simpler Λ setup [7,8]. In the present context the states $|g\rangle$ and $|s_1\rangle$ correspond to the magnetic states with $M_F = -1$ and $M_F = 1$ of the $F = 1$ hyperfine ground level, whereas the state $|s_2\rangle$ represents the hyperfine ground state with $F = 2$ and $M_F = 1$. The two states $|e_1\rangle$ and $|e_2\rangle$ correspond to the electronic excited states with $M_F = 0$ of the $F = 1$ and $F = 2$ manifolds. To make the double tripod setup both probe beams are to be circular σ^+ polarized and all four control beams are to be circular σ^- polarized. Note that such a scheme can be implemented by adding three extra control laser beams to the Λ setup used previously in the experiment by Liu *et al.* [7].

In the previous studies of the multicomponent light [38,39] the counterpropagating probe and control beams have been considered. Here we analyze an opposite situation where the probe and control fields co-propagate (along the z axis). Probe fields can then be written as $\mathcal{E}_1(\mathbf{r},t)e^{ik_1z}$, $\mathcal{E}_2(\mathbf{r},t)e^{ik_2z}$, with $k_j = \omega_j/c$ being the central wave vector of the jth probe beam. The co-propagating setup is more suited for the efficient transfer of an optical vortex between the control and probe beams we are interested in. For paraxial beams the amplitudes $\mathcal{E}_1(\mathbf{r},t)$ and $\mathcal{E}_2(\mathbf{r},t)$ depend weakly on the propagation direction z, the fast spatial dependence being accommodated in the exponential factors e^{ik_jz}. The same applies to the control beams which co-propagate along the z axis and has the form $\Omega_{jq}e^{ik_{jq}z}$, with $k_{jq} = \omega_{jq}/c$ being the central wave vector.

We shall neglect the atomic center-of-mass motion. The electronic properties of the atomic ensemble is described by the atomic flip operators σ_{e_jg} and σ_{s_jg} describing the coherences between the atomic internal states $|e_j\rangle$, $|s_j\rangle$, and $|g\rangle$ at a certain spatial point. For convenience we introduce the column of the probe field amplitudes $\mathcal{E} = (\mathcal{E}_1,\mathcal{E}_2)^T$ and columns of atomic flip operator $\sigma_{eg} = (\sigma_{e_1g},\sigma_{e_2g})^T$ and $\sigma_{sg} = (\sigma_{s_1g},\sigma_{s_2g})^T$. Defining the parameter $g = g_j = \mu_j(\omega_j/2\varepsilon_0\hbar)^{1/2}$ characterizing the atom-light coupling strength (assumed to be equal for both probe fields), the equation for the slowly in time and in space varying amplitudes of the probe fields can be written as follows:

$$\partial_t\mathcal{E} + c\partial_z\mathcal{E} - i\tfrac{1}{2}c\hat{k}^{-1}\nabla_\perp^2\mathcal{E} = ign\sigma_{eg}, \tag{1}$$

where n denotes the atomic density and $\hat{k} = \mathrm{diag}(k_1,k_2)$ is a diagonal 2×2 matrix of the probe field wave vectors.

The diffraction term containing the transverse derivatives $\nabla_\perp^2\mathcal{E}$ can be neglected when the change in the phase of the probe fields due to this term is much smaller than π. The transverse derivative can be estimated as $\nabla_\perp^2\mathcal{E} \sim \sigma^{-2}\mathcal{E}$, where σ is a characteristic transverse dimension of the probe beams. If the probe beam carries an optical vortex, σ can be associated with a width of the vortex core. On the other hand, for the probe beam without an optical vortex, σ is a characteristic width of the beam. The change in time of the probe field can be estimated as $\partial_t\mathcal{E} \sim cL^{-1}\Delta\mathcal{E}$, where L is the length of the atomic cloud and $\Delta\mathcal{E}$ is the change of the field. Thus the change of the phase due to the diffraction term is $L/2k\sigma^2$. It can be neglected when the sample length L is not too large, $L\lambda/\sigma^2 \ll 1$. Taking the length of the atomic cloud $L = 100\ \mu$m, the characteristic transverse dimension of the probe beam $\sigma = 20\ \mu$m, and the wavelength $\lambda = 1\ \mu$m, we obtain $L\lambda/\sigma^2 = 0.25$. Therefore we can drop out the diffraction term in Eq. (1) obtaining

$$\partial_t \mathcal{E} + c\partial_z \mathcal{E} = ign\sigma_{eg}. \tag{2}$$

Equations describing the atom-light coupling are

$$i\partial_t \sigma_{eg} = -i\gamma\sigma_{eg} - \hat{\Omega}\sigma_{sg} - g\mathcal{E}, \tag{3}$$

$$i\partial_t \sigma_{sg} = \hat{\delta}\sigma_{sg} - \hat{\Omega}^\dagger \sigma_{eg}, \tag{4}$$

where the dagger refers to a Hermitian conjugated matrix. The equations are treated in the frames of reference rotating with frequencies ω_j and $\omega_j - \omega_{jj}$, respectively. Here $\hat{\delta} = \mathrm{diag}(\delta_1,\delta_2)$ is a diagonal 2×2 matrix of two photon detunings with $\delta_q = \omega_{s_q g} + \omega_{1q} - \omega_1 = \omega_{s_q g} + \omega_{2q} - \omega_2$, $\hat{\Omega}$ is a 2×2 matrix of Rabi frequencies with matrix elements Ω_{ij}, and γ is the decay rate of excited levels. The decay rate γ is assumed to be the same for both levels $|e_1\rangle$ and $|e_2\rangle$. Initially the atoms are in the ground level g and the Rabi frequencies of the probe fields are considered to be much smaller than those of the control fields. Consequently one can neglect the depletion of the ground level $|g\rangle$.

III. PROPAGATION OF THE PROBE BEAMS

Equations (3) and (4) provide two limiting cases. If the Rabi frequencies of the control beams driving transitions from the level $|s_2\rangle$ are proportional to the Rabi frequencies of the beams driving transitions from the level $|s_1\rangle$ ($\Omega_{22}/\Omega_{21} = \Omega_{12}/\Omega_{11}$), the double tripod system becomes equivalent to a double Λ system for zero two-photon detuning. On the other hand, if $\Omega_{11}\Omega_{21}^* + \Omega_{12}\Omega_{22}^* = 0$, the double tripod system is equivalent to two not connected tripod systems for zero two-photon detuning. We shall concentrate on the case where the double tripod system is not equivalent to a double Λ system and the inverse matrix $(\hat{\Omega}^\dagger)^{-1}$ does exist. Thus Eq. (4) relates σ_{eg} to σ_{sg} as

$$\sigma_{eg} = (\hat{\Omega}^\dagger)^{-1}(\hat{\delta} - i\partial_t)\sigma_{sg}. \tag{5}$$

A. Adiabatic approximation

In what follows the control and probe beams are considered to be close to the two-photon resonance. Application of such resonant beams causes the electromagnetically induced transparency (EIT) in which the optical transitions from the atomic ground states $|g\rangle$, $|s_1\rangle$, and $|s_2\rangle$ interfere destructively preventing population of the excited states $|e_1\rangle$ and $|e_2\rangle$. The adiabatic approximation is obtained neglecting the population of the latter excited states described by the spin-flip operator σ_{eg} in Eq. (3). Thus one has

$$\sigma_{sg} = -g\hat{\Omega}^{-1}\mathcal{E}. \tag{6}$$

Equations (2), (5), and (6) provide a closed set of equations for the electric-field amplitudes $\mathcal{E}_1$ and $\mathcal{E}_2$. Assuming the control beams to be time independent, one arrives at the following matrix equation for the column of the probe fields:

$$(c^{-1} + \hat{v}^{-1})\partial_t \mathcal{E} + \partial_z \mathcal{E} + i\hat{v}^{-1}\hat{D}\mathcal{E} = 0, \tag{7}$$

where

$$\hat{D} = \hat{\Omega}\hat{\delta}\hat{\Omega}^{-1} \tag{8}$$

is a matrix of the two-photon detuning and

$$\hat{v} = \frac{c}{g^2 n}\hat{\Omega}\hat{\Omega}^\dagger \tag{9}$$

is the matrix of group velocity. If the two-photon detunings δ_1 and δ_2 are zero ($\hat{D} = 0$), the last term drops out in the equation of motion (7).

For generality the group velocity matrix $\hat{v}$ is not diagonal and thus the probe fields $\mathcal{E}_1$ and $\mathcal{E}_2$ do not have a definite group velocity. This leads to the mixing between fields $\mathcal{E}_1$ and $\mathcal{E}_2$. We shall return to this issue in the following section.

B. Nonadiabatic corrections

In order to obtain nonadiabatic corrections one needs to include the decay rate γ of the exited levels. Substituting Eq. (5) into Eq. (3) yields

$$\sigma_{sg} = -g\hat{\Omega}^{-1}\mathcal{E} - i\gamma\hat{\Omega}^{-1}(\hat{\Omega}^\dagger)^{-1}(\hat{\delta} - i\partial_t)\sigma_{sg}, \tag{10}$$

where the decay rate γ is assumed to be much larger than the rate of changes of σ_{eg}. Equation (10) can be solved iteratively. Equation (6) is the first-order solution. Substituting expression (6) for σ_{sg} into Eq. (10) one arrives at the second-order solution,

$$\sigma_{sg} = -g\hat{\Omega}^{-1}\mathcal{E} + i\gamma g\hat{\Omega}^{-1}(\hat{\Omega}^\dagger)^{-1}(\hat{\delta} - i\partial_t)\hat{\Omega}^{-1}\mathcal{E}. \tag{11}$$

This leads to a more general equation for the propagation of the probe fields (EIT polaritons) in the atomic cloud,

$$(c^{-1} + \hat{v}^{-1})\partial_t \mathcal{E} + \partial_z \mathcal{E} + i\hat{v}^{-1}\hat{D}\mathcal{E} + \frac{c\gamma}{g^2 n}[\hat{v}^{-1}(\hat{D} - i\partial_t)]^2\mathcal{E} = 0. \tag{12}$$

The last term represents nonadiabatic correction providing a finite lifetime for the polaritons.

IV. TRANSFER OF AN OPTICAL VORTEX

A. Control beams with optical vortices

Up to now no assumption has been made concerning the spatial profile of the control beams. In the following the control beams with Rabi frequencies Ω_{11} and Ω_{22} are assumed to carry optical vortices. Specifically, we take the intensities to be equal $|\Omega_{11}| = |\Omega_{22}|$ and vorticities to be opposite: $l_{11} = -l_{22} \equiv l$. Another two nonvortex control beams also have equal amplitudes, $|\Omega_{12}| = |\Omega_{21}|$, yet there might be a phase difference $2S$ between the fields. Under these conditions the amplitudes of the control beams can be written as

$$\Omega_{11} = |\Omega_{11}|e^{il\varphi}, \quad \Omega_{22} = |\Omega_{11}|e^{-il\varphi}, \tag{13}$$

$$\Omega_{12} = |\Omega_{12}|, \quad \Omega_{21} = |\Omega_{12}|e^{-i2S}. \tag{14}$$

When $S = \pi/2$ and two-photon detunings are zero, two independent tripods are formed all over the space. Furthermore if $|\Omega_{11}| = 0$, two independent tripods are formed for any value of S. This takes place at the axis of the optical vortex. On the other hand, if $S = 0$ and $|\Omega_{11}| = |\Omega_{12}| \neq 0$, one arrives at the double-Λ case for which the inverse velocity matrix becomes singular.

Introducing the angle

$$\tan\phi = \frac{|\Omega_{11}|}{|\Omega_{12}|} \tag{15}$$

and the total Rabi frequency

$$\Omega(\rho) = \sqrt{|\Omega_{12}|^2 + |\Omega_{11}|^2}, \tag{16}$$

the eigenvalues of the velocity matrix have the form

$$v^{\pm} = v_0[1 \pm \cos(S)\sin(2\phi)], \tag{17}$$

with

$$v_0(\rho) = \frac{c\Omega^2}{g^2 n}. \tag{18}$$

Here ρ is the cylindrical radius (the distance from the vortex core). When $S = \pi/2$, the velocity matrix $\hat{v}$ is diagonal and the probe fields are decoupled.

B. Creation of the second probe field with optical vortex

Since the group velocity matrix $\hat{v}$ is not necessarily diagonal, the individual probe fields $\mathcal{E}_1$ and $\mathcal{E}_2$ generally do not have a definite group velocity. Only their combinations $\chi^{\pm}$ for which $\hat{v}\chi^{\pm} = v^{\pm}\chi^{\pm}$ propagate with the definite velocities $v^{\pm}$ in the atomic cloud. If $v^+ = v^-$, this leads to the mixing between probe fields $\mathcal{E}_1$ and $\mathcal{E}_2$.

Suppose that a single component of the monochromatic probe beam $\mathcal{E}_1 = \mathcal{E}_1(0,\rho,\varphi,t) \sim e^{-i\Delta\omega t}$ is incident on the atomic cloud at $z = 0$, with $\Delta\omega$ being the deviation of the frequency of the probe field $\mathcal{E}_1$ from its central frequency ω_1. We use cylindrical coordinates with cylindrical radius ρ, azimuth φ, and longitudinal position z. The atomic gas is considered to be uniform along the propagation direction z form the entry point of the probe beam at $z = 0$ to its exit at $z = L$. At the end of the cloud (at $z = L$) the transmitted fields are $\mathcal{E}_1(L,\rho,\varphi,t) = T_1(\rho,\varphi)\mathcal{E}_1(0,\rho,t)$ and $\mathcal{E}_2(L,\rho,\varphi,t) = T_2(\rho,\varphi)\mathcal{E}_1(0,\rho,t)$, where $T_1(\rho,\varphi)$ and $T_2(\rho,\varphi)$ are the corresponding transmission amplitudes. Equation (12) can be written in the following form for the monochromatic probe fields and the control beams given by Eqs. (13) and (14):

$$\partial_z\mathcal{E} = i(K_0 + K_x\sigma_x + K_y\sigma_y)\mathcal{E}, \tag{19}$$

where σ_x and σ_y are Pauli matrices and

$$K_0 = \frac{\Delta\omega}{c} + \frac{\Delta\omega}{2}\left[\frac{1}{v^+} + \frac{1}{v^-} + i\frac{2L}{\alpha}\Delta\omega\left(\frac{1}{(v^+)^2} + \frac{1}{(v^-)^2}\right)\right], \tag{20}$$

$$K_x = \cos(S + l\varphi)\frac{\Delta\omega}{2}\left[\frac{1}{v^+} - \frac{1}{v^-} + i\frac{2L}{\alpha}\Delta\omega\left(\frac{1}{(v^+)^2} - \frac{1}{(v^-)^2}\right)\right], \tag{21}$$

$$K_y = -\sin(S + l\varphi)\frac{\Delta\omega}{2}\left[\frac{1}{v^+} - \frac{1}{v^-} + i\frac{2L}{\alpha}\Delta\omega\left(\frac{1}{(v^+)^2} - \frac{1}{(v^-)^2}\right)\right]. \tag{22}$$

Here the losses enter via the optical density

$$\alpha = 2\frac{g^2 nL}{c\gamma}. \tag{23}$$

Equation (19) has the plane-wave solutions $\mathcal{E} \sim e^{i\Delta kz}$ with

$$\Delta k = K_0 \pm K_\perp, \tag{24}$$

where

$$K_\perp = \sqrt{K_x^2 + K_y^2}. \tag{25}$$

The spatial development of monochromatic probe fields is described by Eq. (19) providing a formal solution $\mathcal{E}(z) = e^{i(K_0 + K_x\sigma_x + K_y\sigma_y)z}\mathcal{E}(0)$. Thus one can relate the two-component probe field at the entrance and exit points as

$$\mathcal{E}(L) = e^{iK_0L}\left[\cos(K_\perp L) + i\frac{K_x\sigma_x + K_y\sigma_y}{K_\perp}\sin(K_\perp L)\right]\mathcal{E}(0). \tag{26}$$

Since the two-component probe field at the entrance ($z = 0$) is $\mathcal{E}(0) = (1,0)^T$, the transmission amplitudes read

$$T_1 = e^{iK_0L}\cos(K_\perp L), \tag{27}$$

$$T_2 = i\frac{K_x + iK_y}{K_\perp}e^{iK_0L}\sin(K_\perp L). \tag{28}$$

Let us choose the control fields Ω_{11} and Ω_{22} to be the first-order Laguerre-Gaussian beam with $l = 1$, the other two control fields beams being the plane waves, $|\Omega_{12}| = |\Omega_{21}| =$ const. In this case one has

$$\frac{|\Omega_{11}|}{|\Omega_{12}|} = \frac{|\Omega_{22}|}{|\Omega_{21}|} = a\frac{\rho}{\sigma}e^{-\rho^2/\sigma^2}, \tag{29}$$

where ρ is the cylindrical radius (the distance from the vortex core), σ represents the beam width, and a defines the relative strength of the vortex and nonvortex beams. Expanding T_2, the first term in the power series of ρ reads

$$T_2(\rho,\varphi) \approx -2i\frac{\Delta\omega L}{v_0(0)}a\frac{\rho}{\sigma}\cos(S)\exp\left[-il\varphi - iS + i\frac{\Delta\omega L}{c} + i\frac{\Delta\omega L}{v_0(0)}\left(1 + i\frac{2}{\alpha}\frac{\Delta\omega L}{v_0(0)}\right)\right]\left(1 + i\frac{4}{\alpha}\frac{\Delta\omega L}{v_0(0)}\right). \tag{30}$$

Equation (30) shows that the transmission amplitude $T_2(\rho,\varphi)$ increases linearly with the distance ρ and contains a vortex phase factor $-l\varphi$. Thus in a vicinity of the vortex core the generated second beam looks very much like the Laguerre-Gause beam.

In the whole range of distances ρ the transmission probabilities are shown in Fig. 2 for the phase $S = 0$. The nonadiabatic losses are seen to decrease the maximum amplitude of the second probe beam. It is noteworthy that the detuning frequency $\Delta\omega$ and the length L enter the transmission probabilities only in the combination $\Delta\omega L/v_0(0)$. Thus increasing the sample length has the same effect as increasing the detuning.

C. Estimation of the maximum detuning

As can be seen from the last term in Eq. (12), the detuning $\Delta\omega$ introduces a finite life time for the polariton. The last term in Eq. (12) yields the decay rate inversely proportional to the group velocity, therefore, the lifetime of the polariton is determined by the minimum of the group velocity. However, the group velocity cannot be arbitrarily small because of the

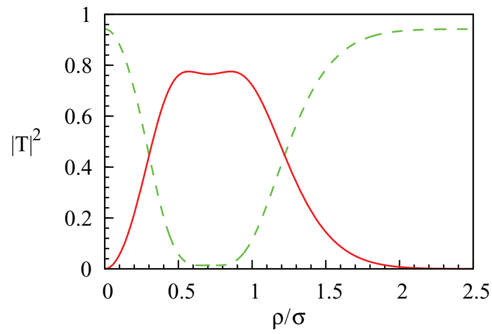

FIG. 2. (Color online) Dependence of the transmission probabilities $|T_1|^2$ (dashed green) and $|T_2|^2$ (solid red) on the dimensionless distance from the vortex core ρ/σ. Transmission probabilities are calculated using Eqs. (27) and (28) for the phase $S = 0$, the parameter $a = 1$, and the optical density $\alpha = 100$. The detuning frequency $\Delta\omega$ is chosen such that the equality $\Delta\omega L[1/v^-(\rho) - 1/v^+(\rho)] = \pi$ holds at the radius $\rho = \sigma/\sqrt{2}$ where the difference between two eigenvalues of group velocity is maximum. This gives $\Delta\omega L/v_0(0) \approx 1.22$. We seek to maximize the difference between eigenvalues because larger difference leads to more effective creation of the second probe beam.

adiabaticity requirement. Thus we assume the minimum group velocity to be of the order of $v_0(\rho)$ at $\rho = 0$, i.e., $v_{\min} \sim v_0(0)$. The lifetime of the polariton is then

$$\tau^{-1} = \gamma[\Delta\omega/\Omega(0)]^2. \tag{31}$$

The requirement that the group velocity should be not too small constrains the parameters of the beams: If $a > \sqrt{2e}$ and $S = 0$ or $S = \pi$, the minimum group velocity is 0.

We can assume that the characteristic time of polariton evolution is the time required to cross the atomic cloud of the length L. In the case of co-propagating control and probe beams the characteristic time is $\tau_{\rm pol} = L/v_{\min} \sim L/v_0(0)$. The characteristic time of the polariton evolution $\tau_{\rm pol}$ should be much smaller than the polariton lifetime τ. From this condition we obtain a constraint on the detuning,

$$\Delta\omega \ll \frac{\Omega(0)}{\sqrt{\gamma\tau_{\rm pol}}} = \Omega(0)\sqrt{\frac{v_0(0)}{\gamma L}}. \tag{32}$$

Equation (32) can be written in the form

$$\frac{2}{\alpha}\left(\frac{\Delta\omega L}{v_0(0)}\right)^2 \ll 1. \tag{33}$$

This condition also follows from the requirement that the last term in Eqs. (20)–(22) should be small compared to other terms.

On the other hand, the optical density α of the atomic cloud is constrained from below. The maximum amplitude of the probe field $\mathcal{E}_2$ is when $\Delta\omega L(1/v^- - 1/v^+) \sim \pi$. Assuming that $(1/v_{\rm gr}^- - 1/v_{\rm gr}^+) \sim 1/v_0(0)$ we get $\Delta\omega \sim \pi v_0(0)/L$. Substituting into Eq. (32) we obtain

$$L \gg \frac{\pi^2\gamma v_0(0)}{\Omega^2(0)}. \tag{34}$$

This condition means that the optical density must be sufficiently large: $\alpha \gg 2\pi^2 \approx 20$.

Note that nonresonant transitions to other hyperfine levels of the electronic excited state can become important when the hyperfine splitting is not large enough compared with the detuning $\Delta\omega$, with the Rabi frequencies of control fields, or with the natural decay rate. Influence of the nonresonant transitions has been studied in Refs. [61,62] showing that the position of EIT resonance is shifted and the medium is no longer perfectly transparent. Thus, influence of the nonresonant transitions makes the losses larger than those shown in Fig. 2.

V. CONCLUDING REMARKS

We have analyzed the manipulation of slow light with the OAM by using control laser beams with and without optical vortices. We have considered a situation where the atom-light interaction represents a double tripod scheme involving four control laser beams of different frequencies which renders medium transparent for a pair of low intensity probe fields. Under the conditions of electromagnetically induced transparency (EIT) the medium supports a lossless propagation of slow light quasiparticles known as dark state polaritons. In the case of double tripod setup these polaritons become two-component (spinor) quasiparticles, involving both probe fields. By properly choosing the control lasers one can generate a tunable coupling between the constituent probe fields. Here we have studied the interaction between the probe fields when two control beams carry optical vortices of opposite helicity. As a result, a transfer of the optical vortex from the control to the probe fields takes place. Notably, the transfer of the optical vortex occurs during the polariton propagation without switching off the control beams. This feature is missing in a single tripod scheme where the optical vortex can be transferred from the control to the probe field only during either the storage or retrieval of light. The manipulation of spinor slow light with the optical vortices has potential application in the optical information processing in quantum atomic gases.

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