Transfer of optical vortices in coherently prepared media

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We consider transfer of optical vortices between laser pulses carrying orbital angular momentum in a cloud of cold atoms characterized by the Λ configuration of the atom-light coupling. The atoms are initially prepared in a coherent superposition of the lower levels, creating a so-called phaseonium medium. If a single vortex beam initially acts on one transition of the scheme, an extra laser beam is subsequently generated with the same vorticity as that of the incident vortex beam. The absorption of the incident probe beam takes place mostly at the beginning of the atomic medium within the absorption length. The losses disappear as the probe beam propagates deeper into the medium where the atoms are transferred to their dark states. The method is extended to a tripod atom-light coupling scheme and a more general $(n+1)$-level scheme containing n ground states and one excited state, allowing for creation of multiple twisted light beams. We also analyze generation of composite optical vortices in the scheme using a superposition of two initial vortex beams and study lossless propagation of such composite vortices.

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I. INTRODUCTION

The coherent manipulation of pulse propagation through atomic ensembles [1–4] leads a plethora of important phenomena, such as electromagnetically induced transparency (EIT) [1,5–7] and slow light propagation [1,5,6,8], enhancement of optical nonlinearities [9–12], generation of matched pulses [13,14], creation of a spinor slow light [15–19], and formation of adiabatons [20,21] and optical solitons [22,23]. It has been demonstrated that due to the EIT the light pulses not only could be slowed down, but also stored by switching off the controlled beam [5,24–27]. Therefore, the atomic system can be used as an optical memory for transferring the quantum state of light to the matter and back to the light [5,28,29]. By using extra energy levels and additional laser field one arrives at more complex atom-light coupling schemes [2,30–41] which can provide more than one dark state and offer different directions in studying of propagation effects in coherently driven atomic media.

Light can carry an orbital angular momentum (OAM) [42,43]. Such light beams have helical (twisted) wavefronts that spiral along the beam direction much like a corkscrew. The twisted light field is characterized by a phase factor $e^{il\phi}$, where ϕ is the azimuthal angle with respect to the beam axis and l denotes the vortex integer winding number (OAM number). When interacting with atoms such optical vortex beams reveal a number of interesting effects, including light-induced torque [44,45], atom vortex beams [46], entanglement of OAM states of photon pairs [47], OAM-based four-wave mixing [48,49], spatially dependent optical transparency [50–52], and the vortex slow light [33,53–55]. The twisted slow light [18,33,53–57] gives additional possibilities in manipulation of the optical information during the storage and retrieval of the slow light [58,59].

The previous studies on the EIT have concentrated on a situation where the atoms are initially in their ground states, and the Rabi frequency of the probe field is much weaker than that of the control field. It has been demonstrated that the OAM of the control vortex beam can be transferred to the probe field in the tripod atom-light coupling scheme during the storage and retrieval of the probe field [33,57]. Without switching off and on of the control fields (hence without storage and retrieval of the probe field), transfer of optical vortices take place by applying a pair of weaker probe fields in the closed loop double-Λ [55] or double-tripod [18] schemes.

In this paper, we demonstrate that the exchange of optical vortices in nonclosed loop structures is possible under the condition of weak atom-light interaction in coherently prepared atomic media. To this end, we analyze the interaction of multicomponent laser pulses carrying OAM propagating in multilevel atom-light coupling schemes with atoms prepared in a coherent superposition of lower levels. Such a medium has been named the phaseonium [60–63]. We derive the basic equations describing the propagation of the coherent laser pulses weakly interacting with atoms in multilevel configurations. To elucidate the physical situation of exchange of OAMs, we begin with a basic three-level configuration, the Λ system, containing only a pair of laser pulses. Subsequently we extend our model to more complicated schemes involving additional laser pulses and additional atomic levels. It is shown that the transfer of optical vortices is also possible for the tripod system and a more general $(n+1)$-level scheme. Furthermore we show that composite optical vortices can be formed in the Λ system when both probe fields are present at entrance of the medium.

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The phaseonium medium proposed in this paper is based on a coherent superposition of the ground states and can be realized experimentally using the fractional or partial stimulated Raman adiabatic passage (STIRAP) [64]. The generation of a quantum superposition of ground states in a robust and controlled way is known to be possible in a four-state tripod system by using a sequence of three laser pulses [30,31]. Such a technique is based on the existence of two degenerate dark states and their interaction. The mixing of the dark states can be controlled by changing the relative delay of the pulses, and thus an arbitrary superposition state can be created. This method for creation of coherent superpositions can be generalized to N level schemes.

The method described here for transfer of optical vortices may find application for creation of structured light by another light [53]. Using our method one could create a vortex at a wavelength for which it is not possible to do it directly with standard optics (e.g., far infrared or UV) [65]. In addition, the transfer of vortices is a possible tool for manipulation of information encoded into OAM of light.

II. THREE-LEVEL Λ SYSTEM

Let us first consider the Λ scheme for the transfer of optical vortices, illustrated in Fig. 1. Specifically, we study the propagation of two laser pulses with the Rabi frequencies Ω_1 and Ω_2 (pulse pair) in a medium consisting of atoms in the three-level Λ configuration of the atom-light coupling. The two atomic lower states $|g_1\rangle$ and $|g_2\rangle$ are coupled to an excited state $|e\rangle$ via the two light fields. The Hamiltonian for such a system reads in the appropriate rotating frame and in the interaction picture

$$H_\Lambda = \Omega_1|g_1\rangle\langle e| + \Omega_2|g_2\rangle\langle e| + \text{H.c.} \tag{1}$$

The dynamics of the pulse pair Ω_1 and Ω_2 and two atomic coherences ρ_{g_1e} and ρ_{g_2e} are described by the Maxwell-Bloch equations (MBE) for an open system:

$$\dot\rho_{g_1e} = i(\delta_1 + i\gamma_{eg_1})\rho_{g_1e} - i\Omega_1(\rho_{ee} - \rho_{g_1g_1}) + i\Omega_2\rho_{g_1g_2}, \tag{2}$$

$$\dot\rho_{g_2e} = i(\delta_2 + i\gamma_{eg_2})\rho_{g_2e} - i\Omega_2(\rho_{ee} - \rho_{g_2g_2}) + i\Omega_1\rho_{g_2g_1}, \tag{3}$$

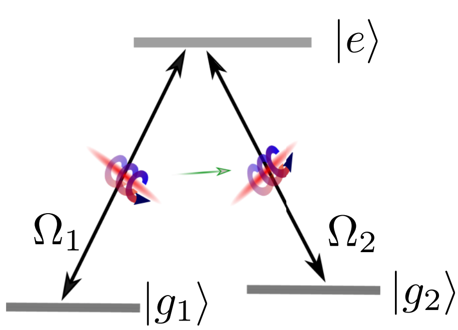

FIG. 1. Schematic diagram of the three-level Λ quantum system containing an upper state $|e\rangle$ and lower levels $|g_1\rangle$ and $|g_2\rangle$ interacting with two Rabi frequencies Ω_1 and Ω_2.

and

$$\frac{\partial\Omega_1}{\partial z} + c^{-1}\frac{\partial\Omega_1}{\partial t} = i\frac{\alpha_1\gamma_{eg_1}}{2L}\rho_{g_1e}, \tag{4}$$

$$\frac{\partial\Omega_2}{\partial z} + c^{-1}\frac{\partial\Omega_2}{\partial t} = i\frac{\alpha_2\gamma_{eg_2}}{2L}\rho_{g_2e}, \tag{5}$$

where α_1 and α_2 are the optical depths of both laser pulses Ω_1 and Ω_2, L denotes the optical length of the medium, and γ_{eg_1} and γ_{eg_2} are the rates of decay from the excited state $|e\rangle$ to lower states $|g_1\rangle$ and $|g_2\rangle$, respectively. We have defined the detunings as $\delta_1 = \omega_{eg_1} - \omega_1$ and $\delta_2 = \omega_{eg_2} - \omega_2$, where ω_{eg_1} and ω_{eg_2} are the frequencies of the transitions $|g_1\rangle \leftrightarrow |e\rangle$ and $|g_2\rangle \leftrightarrow |e\rangle$, respectively, while ω_1 and ω_2 represent the central frequencies of the probe beams. We have disregarded the diffraction terms containing the transverse derivatives $(2k_1)^{-1}\nabla_\perp^2\Omega_1$ and $(2k_2)^{-1}\nabla_\perp^2\Omega_2$ in the Maxwell equations (4) and (5), where $k_1 = \omega_1/c$ and $k_2 = \omega_2/c$ are the central wave vectors of the first and second beams. One can evaluate these terms as $\nabla_\perp^2\Omega_{1(2)} \sim w^{-2}\Omega_{1(2)}$, where w represents a characteristic transverse dimension of the laser beams. This can be a width of the vortex core if the beam carries an optical vortex or a characteristic width of the beam if it has no vortex. Consequently the change of the phase of the probe beams due to the diffraction term after passing the medium is estimated to be $L/2kw^2$, where L is the length of the atomic cloud, where $k \approx k_{1(2)}$. The phase change $L/2kw^2$ can be neglected when the sample length L is not too large, $L\lambda/w^2 \ll \pi$, where $\lambda = 2\pi/k$ is an optical wavelength. For example, by taking the length of the atomic cloud to be $L = 100\ \mu$m, the characteristic transverse dimension of the beams $w = 20\ \mu$m, and the wavelength $\lambda = 1\ \mu$m, we obtain $L\lambda/w^2 = 0.25$. Under these conditions the diffraction terms do not play a significant role and we can drop it out in Eqs. (4) and (5).

Let us assume that the atoms are initially in a superposition of both lower levels (the phaseonium medium)

$$|\psi(0)\rangle = c_1|g_1\rangle + c_2|g_2\rangle. \tag{6}$$

We consider a weak atom-light interaction where $|\Omega_1|, |\Omega_2| \ll \gamma_{eg_1}, \gamma_{eg_2}$. Then, to the first order one has $\rho_{ee} \approx 0$, $\rho_{g_1g_1} \approx |c_1|^2$, $\rho_{g_2g_2} \approx |c_2|^2$, and $\rho_{g_1g_2} \approx c_1c_2^*$, giving the following the steady-state solutions for the coherences ρ_{g_1e} and ρ_{g_2e}:

$$\rho_{g_1e} = -\frac{|c_1|^2\Omega_1 + c_1c_2^*\Omega_2}{\delta_1 + i\gamma_{eg_1}}, \tag{7}$$

$$\rho_{g_2e} = -\frac{c_1^*c_2\Omega_1 + |c_2|^2\Omega_2}{\delta_2 + i\gamma_{eg_2}}. \tag{8}$$

The first-order approximation is valid when $|\rho_{g_1e}|, |\rho_{g_2e}| \ll 1$. Otherwise we cannot assume that $\rho_{g_1g_1}$ and $\rho_{g_1g_2}$ are not changing during the propagation of light.

Substituting Eqs. (7) and (8) into the Maxwell equations (4) and (5) one arrives at the following coupled equations for the propagation of the pulse pair [41]:

$$\frac{\partial\Omega_1}{\partial z} = -i\beta_1(|c_1|^2\Omega_1 + c_1c_2^*\Omega_2), \tag{9}$$

$$\frac{\partial\Omega_2}{\partial z} = -i\beta_2(c_1^*c_2\Omega_1 + |c_2|^2\Omega_2), \tag{10}$$

where

$$\beta_a = \frac{\alpha_a \gamma_{eg_a}}{2L(\delta_a + i\gamma_{eg_a})}, \tag{11}$$

with $a = 1, 2$.

The second laser field is assumed to be zero $\Omega_2(0) = 0$ at the entrance $z = 0$, while $\Omega_1(0) = \Omega$. Under these conditions the solutions to Eqs. (9) and (10) read

$$\Omega_1(z) = \frac{\Omega}{X_2}(\beta_1|c_1|^2 e^{-iX_2 z} + \beta_2|c_2|^2), \tag{12}$$

$$\Omega_2(z) = \frac{\Omega}{X_2} c_1^* c_2 \beta_2 (e^{-iX_2 z} - 1), \tag{13}$$

where

$$X_2 = \beta_1|c_1|^2 + \beta_2|c_2|^2. \tag{14}$$

Up to now no assumption has been made concerning the spatial profile of the laser fields. We take now that the incident beam Ω_1 has an optical vortex

$$\Omega_1(0) = \Omega = |\Omega|e^{il\phi}, \tag{15}$$

where l is the orbital angular momenta along the propagation axis z and ϕ is the azimuthal angle. For a doughnut Laguerre-Gaussian (LG) beam the transverse profile reads

$$|\Omega| = \varepsilon \left(\frac{r}{w}\right)^{|l|} e^{-r^2/w^2}, \tag{16}$$

where r describes a cylindrical radius, w is a beam waist, and ε represents the strength of the vortex beam. According to Eqs. (12)–(15), the generated pulse beam $\Omega_2(z) \sim e^{il\phi}$ acquires the same phase as the first vortex beam. Therefore, the laser beam Ω_1 transfers its vortex to the generated beam Ω_2.

Equations (12) and (13) show that both light beams experience losses during their propagation. Yet the losses appear only at the entrance of the medium before the EIT is established for both fields. To simplify the discussion, let us take $\alpha_1 = \alpha_2 = \alpha$ and $\gamma_{eg_1} = \gamma_{eg_2} = \gamma$, and consider a situation where both laser fields Ω_1 and Ω_2 are in an exact resonance with the corresponding atomic transitions ($\delta_1 = \delta_2 = 0$). Then Eqs. (11) and (14) lead to $\beta_1 = \beta_2 = X_2 = \frac{1}{2iL_{abs}}$, where $L_{abs} = L/\alpha$ is the absorption length. If optical density of the resonant medium is sufficiently large $\alpha \gg 1$, the absorption length constitutes a fraction of the whole medium $L_{abs} \ll L$. For the distances z exceeding the absorption length $z \gg L_{abs}$ both exponential terms vanish in Eqs. (12) and (13) and the EIT is established leading to lossless propagation of both fields. Using Eqs. (15) and (16) we get

$$\Omega_1(z \gg L_{abs}) = \varepsilon \left(\frac{r}{w}\right)^{|l|} e^{-r^2/w^2}(1 - |c_1|^2)e^{il\phi}, \tag{17}$$

$$\Omega_2(z \gg L_{abs}) = -\varepsilon \left(\frac{r}{w}\right)^{|l|} e^{-r^2/w^2} c_1^* c_2 e^{il\phi}. \tag{18}$$

In this way, the beams experience no absorption loss for large propagation distances $z \gg L_{abs}$. This is illustrated in Fig. 2 showing the dependence of the intensities $|\Omega_1(z)|^2/|\Omega_1(0)|^2$ and $|\Omega_2(z)|^2/|\Omega_1(0)|^2$ given by Eqs. (12) and (13) on the dimensionless distance z/L_{abs} for the resonance case $\delta_1 = \delta_2 = 0$ and $\alpha = 20$. Although initially at the beginning of the atomic medium losses occur, going deeper through the medium the losses disappear as the atoms go to their dark state [6]

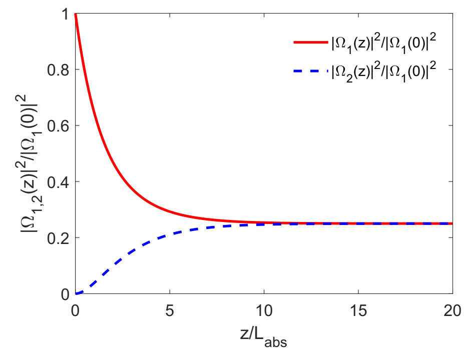

FIG. 2. Dependence of the dimensionless intensities of the light fields $|\Omega_1(z)|^2/|\Omega_1(0)|^2$ and $|\Omega_2(z)|^2/|\Omega_1(0)|^2$ given in Eqs. (12) and (13) on the dimensionless distance z/L_{abs} for $c_1 = c_2 = \frac{1}{\sqrt{2}}$, $\delta_1 = \delta_2 = 0$, and $\alpha = 20$.

$$D(z \gg L_{abs}) = \frac{\Omega_2(z \gg L_{abs})|g_1\rangle - \Omega_1(z \gg L_{abs})|g_2\rangle}{\sqrt{\Omega_1^2(z \gg L_{abs}) + \Omega_2^2(z \gg L_{abs})}}. \tag{19}$$

Let us investigate how sensitive is the proposed method for transferring of optical vortices to errors in the amplitudes and the phases of the superpositions. The sensitivity of system to the errors is given by the derivative of the fields in the output given by Eqs. (12) and (13) with respect to the coefficients c_1 and c_2. Assuming $\beta_1 = \beta_2 = \beta = \frac{1}{2iL_{abs}}$ and using the fact $|c_2| = \sqrt{1 - |c_1|^2}$, Eqs. (12) and (13) can be rewritten as

$$\Omega_1(z) = \Omega + \Omega\big(e^{-i\frac{z}{2iL_{abs}}} - 1\big)|c_1|^2, \tag{20}$$

$$\Omega_2(z) = \Omega\big(e^{-i\frac{z}{2iL_{abs}}} - 1\big)|c_1|\sqrt{1 - |c_1|^2}e^{i\phi_c}, \tag{21}$$

where $\phi_c = \phi_{c_2} - \phi_{c_1}$ is the relative phase of coefficients c_1 and c_2. The relative phase of the coefficients c_1 and c_2 appears only in Eq. (21). Calculating the derivative of the fields given by Eqs. (20) and (21) with respect to the amplitude $|c_1|$ as well as the relative phase ϕ_c gives

$$\frac{\partial\Omega_1(z)}{\partial|c_1|} = 2\Omega|c_1|\big(e^{-i\frac{z}{2iL_{abs}}} - 1\big), \tag{22}$$

$$\frac{\partial\Omega_2(z)}{\partial|c_1|} = \Omega e^{i\phi_c}\big(e^{-i\frac{z}{2iL_{abs}}} - 1\big)\frac{1 - 2|c_1|^2}{\sqrt{1 - |c_1|^2}}, \tag{23}$$

$$\frac{\partial\Omega_2(z)}{\partial\phi_c} = i\Omega_2(z). \tag{24}$$

Equations (22)–(24) show that the proposed method is not very sensitive to the errors in the coefficients. This can be also seen from Eqs. (17) and (18) for $z \gg L_{abs}$. It is clear that the ratio $|\Omega_1|/|\Omega_2|$ is proportional to $|c_2|/|c_1|$. Errors in the amplitudes will change this ratio and, consequently, the intensity of the transferred vortex. The intensity does not depend on the phases of the superpositions. Only the relative phase of the coefficients c_1 and c_2 enters into Eq. (13) or Eq. (18) and changes the global phase of the second field $\Omega_2(z)$.

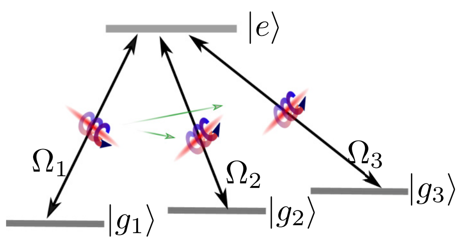

FIG. 3. Schematic diagram of the four-level tripod quantum system containing an upper state $|e\rangle$ and lower levels $|g_1\rangle$, $|g_2\rangle$, and $|g_3\rangle$ interacting with three Rabi frequencies Ω_1, Ω_2, and Ω_3.

III. TRIPOD SYSTEM

Consider next the propagation of three light pulses through a medium consisting of atoms with a tripod level structure shown in Fig. 3. Previously the tripod scheme was employed when studying the transfer of optical vortices [33,57]. It was shown that the transfer of optical vortices from a control field of larger intensity to a probe field of weaker intensity in tripod scheme was only possible through switching off and on of the control laser beams [33,57]. In the following, we demonstrate that the exchange of optical vortices in tripod scheme is possible even without switching off and on of the control beams, if one uses a coherently prepared atomic system.

In the tripod scheme an excited state $|e\rangle$ is coupled to three lower levels $|g_1\rangle$, $|g_2\rangle$, and $|g_3\rangle$ through three laser pulses Ω_1, Ω_2, and Ω_3, respectively. The Hamiltonian for the tripod scheme reads in the interaction representation

$$H_{\mathrm{T}} = \Omega_1|g_1\rangle\langle e| + \Omega_2|g_2\rangle\langle e| + \Omega_3|g_3\rangle\langle e| + \mathrm{H.c.} \tag{25}$$

The MBEs describing the evolution of system can be written as

$$\dot{\rho}_{g_1e} = i(\delta_1 + i\gamma_{eg_1})\rho_{g_1e} - i\Omega_1(\rho_{ee} - \rho_{g_1g_1}) + i\Omega_2\rho_{g_1g_2} + i\Omega_3\rho_{g_1g_3}, \tag{26}$$

$$\dot{\rho}_{g_2e} = i(\delta_1 + i\gamma_{eg_2})\rho_{g_2e} - i\Omega_2(\rho_{ee} - \rho_{g_2g_2}) + i\Omega_1\rho_{g_2g_1} + i\Omega_3\rho_{g_2g_3}, \tag{27}$$

$$\dot{\rho}_{g_3e} = i(\delta_1 + i\gamma_{eg_3})\rho_{g_3e} - i\Omega_3(\rho_{ee} - \rho_{g_3g_3}) + i\Omega_1\rho_{g_3g_1} + i\Omega_2\rho_{g_3g_2}, \tag{28}$$

and

$$\frac{\partial\Omega_1}{\partial z} + c^{-1}\frac{\partial\Omega_1}{\partial t} = i\frac{\alpha_1\gamma_{eg_1}}{2L}\rho_{g_1e}, \tag{29}$$

$$\frac{\partial\Omega_2}{\partial z} + c^{-1}\frac{\partial\Omega_2}{\partial t} = i\frac{\alpha_2\gamma_{eg_2}}{2L}\rho_{g_2e}, \tag{30}$$

$$\frac{\partial\Omega_3}{\partial z} + c^{-1}\frac{\partial\Omega_3}{\partial t} = i\frac{\alpha_3\gamma_{eg_3}}{2L}\rho_{g_3e}, \tag{31}$$

where the diffraction terms have been neglected in the Maxwell equations (29) and (31), like for the Λ scheme.

The atoms are initially prepared in a superposition of three lower states:

$$|\psi(0)\rangle = \mathrm{c}_1|g_1\rangle + c_2|g_2\rangle + c_3|g_3\rangle. \tag{32}$$

For a sufficiently weak atom-light interaction, $|\Omega_j| \ll \gamma_{eg_j}$, we can approximate $\rho_{ee} \approx 0$, $\rho_{g_1g_1} \approx |c_1|^2$, $\rho_{g_2g_2} \approx |c_2|^2$, $\rho_{g_3g_3} \approx |c_3|^2$, $\rho_{g_1g_2} \approx c_1c_2^*$, $\rho_{g_1g_3} \approx c_1c_3^*$, and $\rho_{g_2g_3} \approx c_2c_3^*$, giving the following steady-state equations for the atomic coherences:

$$\rho_{g_1e} = -\frac{|c_1|^2\Omega_1 + c_1c_2^*\Omega_2 + c_1c_3^*\Omega_3}{\delta_1 + i\gamma_{eg_1}}, \tag{33}$$

$$\rho_{g_2e} = -\frac{c_1^*c_2\Omega_1 + |c_2|^2\Omega_2 + c_2c_3^*\Omega_3}{\delta_2 + i\gamma_{eg_2}}, \tag{34}$$

$$\rho_{g_3e} = -\frac{c_1^*c_3\Omega_1 + c_2^*c_3\Omega_2 + |c_3|^2\Omega_3}{\delta_3 + i\gamma_{eg_3}}. \tag{35}$$

Substituting Eqs. (33)–(35) into the Maxwell equations (29)–(31) and assuming that at the entrance ($z = 0$) there is a single light beam $\Omega_1(0) = \Omega$, $\Omega_2(0) = 0$, and $\Omega_3(0) = 0$, we obtain [40,41]

$$\Omega_1(z) = \frac{\Omega}{X_3}(\beta_1|c_1|^2e^{-iX_3z} + \beta_2|c_2|^2 + \beta_3|c_3|^2), \tag{36}$$

$$\Omega_2(z) = \frac{\Omega}{X_3}c_1^*c_2\beta_2(e^{-iX_3z} - 1), \tag{37}$$

$$\Omega_3(z) = \frac{\Omega}{X_3}c_1^*c_3\beta_3(e^{-iX_3z} - 1), \tag{38}$$

where

$$X_3 = \beta_1|c_1|^2 + \beta_2|c_2|^2 + \beta_3|c_3|^2, \tag{39}$$

and β_a is defined by Eq. (11), with $a = 1, 2, 3$. Considering again the first laser pulse Ω_1 initially carries an optical vortex [defined by Eqs. (15) and (16)], two vortex beams $\Omega_2 \sim e^{il\phi}$ and $\Omega_3 \sim e^{il\phi}$ are generated with the same vorticity as the first laser pulse $\Omega_1 \sim e^{il\phi}$.

Using Eqs. (15)–(16) and (36)–(38), one gets for sufficiently large z ($z \gg L_{abs}$) under the resonance condition $\delta_1 = \delta_2 = \delta_3 = 0$ and assuming that $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$, $\gamma_{eg_1} = \gamma_{eg_2} = \gamma_{eg_3} = \gamma$:

$$\Omega_1(z \gg L_{abs}) = \varepsilon\left(\frac{r}{w}\right)^{|l|} e^{-r^2/w^2}(1 - |c_1|^2)e^{il\phi}, \tag{40}$$

$$\Omega_2(z \gg L_{abs}) = -\varepsilon\left(\frac{r}{w}\right)^{|l|} e^{-r^2/w^2}c_1^*c_2e^{il\phi}, \tag{41}$$

$$\Omega_3(z \gg L_{abs}) = -\varepsilon\left(\frac{r}{w}\right)^{|l|} e^{-r^2/w^2}c_1^*c_3e^{il\phi}. \tag{42}$$

Thus the lossless propagation of generated vortex beams takes place at distances exceeding the absorption length L_{abs} as illustrated in Fig. 4. In that case the atomic system goes to a superposition of two dark states [30,31,66,67]:

$$D_1(z \gg L_{abs}) = \frac{\Omega_3(z \gg L_{abs})|g_1\rangle - \Omega_1(z \gg L_{abs})|g_3\rangle}{\sqrt{\Omega_1^2(z \gg L_{abs}) + \Omega_2^2(z \gg L_{abs})}}, \tag{43}$$

$$\begin{aligned} D_2(z \gg L_{abs}) &= [\Omega_1(z \gg L_{abs})\Omega_2(z \gg L_{abs})|g_1\rangle \\ &\quad + \Omega_2(z \gg L_{abs})\Omega_3(z \gg L_{abs})|g_3\rangle \\ &\quad - (\Omega_1^2(z \gg L_{abs}) + \Omega_3^2(z \gg L_{abs}))|g_2\rangle] \end{aligned}$$

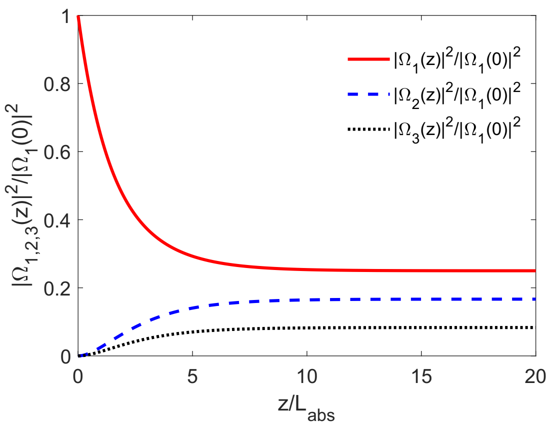

FIG. 4. Dependence of the dimensionless intensities $|\Omega_1(z)|^2/|\Omega_1(0)|^2$, $|\Omega_2(z)|^2/|\Omega_1(0)|^2$, and $|\Omega_3(z)|^2/|\Omega_1(0)|^2$ given by Eqs. (36)–(38) on the dimensionless distance z/L_{abs} for $c_1 = \frac{1}{\sqrt{2}}$, $c_2 = \frac{1}{\sqrt{3}}$, $c_2 = \frac{1}{\sqrt{6}}$, $\delta_1 = \delta_2 = \delta_3 = 0$, and $\alpha = 20$.

$$\times \left[\left(\Omega_1^2(z \gg L_{abs}) + \Omega_3^2(z \gg L_{abs})\right)\left(\Omega_1^2(z \gg L_{abs}) + \Omega_2^2(z \gg L_{abs}) + \Omega_3^2(z \gg L_{abs})\right)\right]^{-1/2}. \quad (44)$$

IV. MULTILEVEL SYSTEM

Let us now extend our model by considering the propagation of n-component light pulses through an $(n+1)$-state atomic medium with n lower atomic states and one excited state shown in Fig. 5. Denoting the excited state by $|e\rangle$, the lower levels by $|g_1\rangle, |g_2\rangle, \ldots, |g_n\rangle$ and the Rabi frequency of laser pulses by Ω_m $(m = 1, 2, \ldots, n)$, the interaction Hamiltonian for such a multilevel atom reads

$$H_{\rm M} = \sum_{m=1}^{n} \Omega_m |g_m\rangle\langle e| + \text{H.c.} \quad (45)$$

The MBEs describing the dynamics of the system are given by

$$\dot{\rho}_{g_m e} = i(\delta_m + i\gamma_{eg_m})\rho_{g_m e} - i\Omega_m(\rho_{ee} - \rho_{g_m g_m}) + i \sum_{j=1; j\neq m}^{n} \Omega_j \rho_{g_m g_j} \quad (46)$$

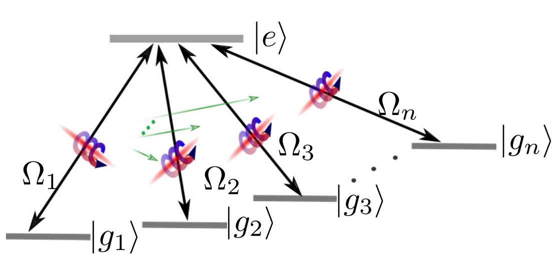

FIG. 5. Schematic diagram of the multilevel quantum system containing an upper state $|e\rangle$ and lower levels $|g_1\rangle$, $|g_2\rangle$, ..., $|g_n\rangle$ interacting with Rabi frequencies Ω_1, Ω_2, ..., Ω_n.

and

$$\frac{\partial \Omega_m}{\partial z} + c^{-1}\frac{\partial \Omega_m}{\partial t} = i\frac{\alpha_m \gamma_{eg_m}}{2L}\rho_{g_m e} \quad (m = 1, 2, \ldots, n), \quad (47)$$

with $m = 1, 2, \ldots, n$. As before, the diffraction terms are ignored in the Maxwell equations (47).

The atoms comprising the system are initially in the superposition of n ground states:

$$|\psi(0)\rangle = \sum_{m=1}^{n} c_m |g_m\rangle. \quad (48)$$

Assuming the atom-light interaction to be sufficiently weak, $|\Omega_j| \ll \gamma_{eg_j}$, one can approximate $\rho_{ee} \approx 0$, $\rho_{g_s g_s} \approx |c_s|^2$, and $\rho_{g_s g_t} \approx c_s c_t^*$ to the first order in all laser fields, giving

$$\rho_{g_m e} = -\frac{\sum_{j=1}^{n} c_m c_j^* \Omega_j}{\delta_m + i\gamma_{eg_m}}, \quad m = 1, 2, \ldots, n. \quad (49)$$

If all the laser pulses except the first one are zero at the entrance $[\Omega_1(0) = \Omega$, while $\Omega_2(0) = 0, \Omega_3(0) = 0, \ldots, \Omega_n(0) = 0]$, the solutions to the MBEs (46) and (47) are [41]

$$\Omega_1(z) = \frac{\Omega}{X_n}\left(\beta_1 |c_1|^2 e^{-iX_n z} + \sum_{N=2}^{n} \beta_N |c_N|^2\right), \quad (50)$$

$$\Omega_N(z) = \frac{\Omega}{X_n} c_1^* c_N \beta_N (e^{-iX_n z} - 1) \quad (N = 2, \ldots, n), \quad (51)$$

with

$$X_n = \sum_{m=1}^{n} \beta_m |c_m|^2. \quad (52)$$

From Eqs. (15) and (16) it follows then that if the first light pulse photons carry an OAM of $\hbar l$ along the propagation direction, $n-1$ optical vortices are generated in the medium with the same vorticity as the first laser beam Ω_1. In the vicinity of the vortex core the generated vortex beams look like a LG beam with their intensity vanishing at the core $r \to 0$.

Calling on Eqs. (15) and (16), Eqs. (50) and (51) provide the following solutions for the distances exceeding the absorption length $z \gg L_{abs}$:

$$\Omega_1(z \gg L_{abs}) = \varepsilon \left(\frac{r}{w}\right)^{|l|} e^{-r^2/w^2}(1 - |c_1|^2)e^{il\phi}, \quad (53)$$

$$\Omega_N(z \gg L_{abs}) = -\varepsilon \left(\frac{r}{w}\right)^{|l|} e^{-r^2/w^2} c_1^* c_N e^{il\phi}, \quad (54)$$

where we have assumed that $\alpha_1 = \alpha_2 = \cdots = \alpha_n = \alpha$, $\gamma_{eg_1} = \gamma_{eg_2} = \cdots = \gamma_{eg_n} = \gamma$, and $\delta_1 = \delta_2 = \cdots = \delta_n = 0$. Equations (53) and (54) demonstrate the lossless propagation of the n-component optical vortices because for $z \gg L_{abs}$ the multilevel model goes to a linear superposition of $n-1$ dark states.

V. COMPOSITE VORTICES

Let us next consider a situation where the Λ scheme is initially prepared in a superposition state given by Eqs. (6), but both fields Ω_1 and Ω_2 are incident on the medium. With

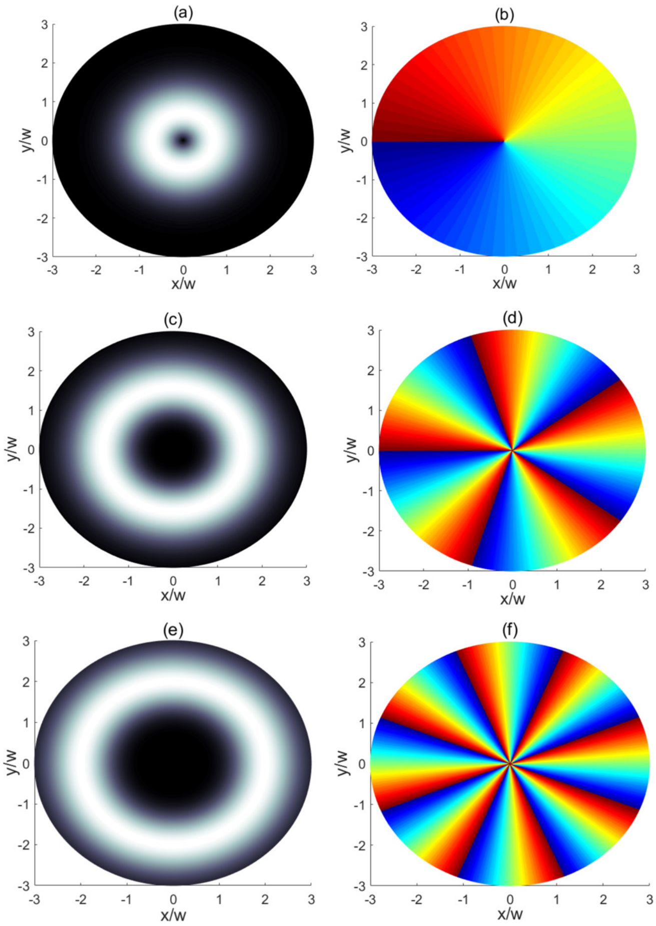

FIG. 6. (a),(c),(e) Intensity distributions in arbitrary units as well as (b),(d),(f) the corresponding helical phase patterns of the beam $\Omega_1(z)$ defined by Eq. (55) generated by combining two vortex beams with vorticities (a),(b) $l_1 = l_2 = 1$, (c),(d) $l_1 = l_2 = 5$, and (e),(f) $l_1 = l_2 = 8$. Here, the parameters are $z = L/2$, $c_1 = c_2 = \frac{1}{\sqrt{2}}$, $\delta_1 = \delta_2 = 0$, $\varepsilon_1 = \varepsilon_2$, and $\alpha = 20$. The intensity distribution and phase pattern of the field $\Omega_2(z)$ are identical to the intensity distribution and phase pattern of the field $\Omega_1(z)$ shown in this figure.

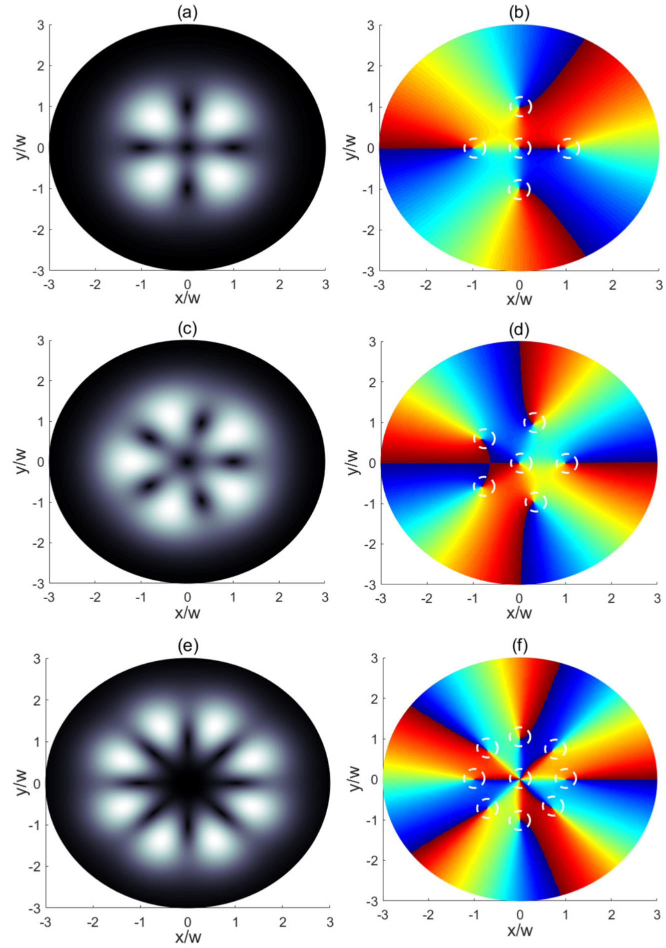

FIG. 7. (a),(c),(e) Intensity distributions in arbitrary units as well as the (b),(d),(f) corresponding helical phase patterns of the beam $\Omega_1(z)$ defined by Eq. (55) generated by combining two vortex beams with vorticities (a),(b) $l_1 = 1$, $l_2 = -3$, (c),(d) $l_1 = -1$, $l_2 = 4$, and (e),(f) $l_1 = 3$, $l_2 = -5$. The selected parameters are the same as in Fig. 6. The white dash lines in the phase patterns show the position of vortices. The intensity distribution and phase pattern of the field $\Omega_2(z)$ are identical to the intensity distribution and phase pattern of the field $\Omega_1(z)$ shown in this figure.

the initial conditions where both incident fields are the vortex beams $\Omega_1(0) = \Omega_{10} = \varepsilon_1(\frac{r}{w})^{|l_1|}e^{-r^2/w^2}e^{il_1\phi}$ and $\Omega_2(0) = \Omega_{20} = \varepsilon_2(\frac{r}{w})^{|l_2|}e^{-r^2/w^2}e^{il_2\phi}$, the solutions to Eqs. (9) and (10) take the form

$$\Omega_1(z) = \frac{1}{X_2}\left[\varepsilon_1\left(\frac{r}{w}\right)^{|l_1|}e^{-r^2/w^2}(\beta_1|c_1|^2e^{-iX_2z} + \beta_2|c_2|^2)e^{il_1\phi} + \varepsilon_2\left(\frac{r}{w}\right)^{|l_2|}e^{-r^2/w^2}c_1c_2^*\beta_1(e^{-iX_2z} - 1)e^{il_2\phi}\right], \quad (55)$$

$$\Omega_2(z) = \frac{1}{X_2}\left[\varepsilon_1\left(\frac{r}{w}\right)^{|l_1|}e^{-r^2/w^2}c_1^*c_2\beta_2(e^{-iX_2z} - 1)e^{il_1\phi} + \varepsilon_2\left(\frac{r}{w}\right)^{|l_2|}e^{-r^2/w^2}(|c_2|^2\beta_2e^{-iX_2z} + |c_1|^2\beta_1)e^{il_2\phi}\right]. \quad (56)$$

In this way by applying two incident vortex beams $\Omega_1(0)$ and $\Omega_2(0)$ one produces two new beams $\Omega_1(z)$ and $\Omega_2(z)$ which may contain different vortices depending on the relative amplitude and phase of the incident beams.

Various situations can appear for the beams created in this way. If the winding numbers of the incident pulses are the same, $l_1 = l_2 = l$, the resulting beams $\Omega_1(z)$ and $\Omega_2(z)$ have the same vorticity l, and the vortex width increases with increasing the winding number l, as illustrated in Figs. 6(a), 6(c), and 6(e). Figures 6(b), 6(d), and 6(f) show the corresponding phase profile of the beams. If $|l_1| < |l_2|$, the resulting composite twisted beam contains a vortex of charge l_1 located at the beam center which is surrounded by $|l_1 - l_2|$ peripheral vortices (Fig. 7). In this case, two light vortices with different winding numbers l_1 and l_2 around the same axis result in formation of vortices with shifted axes. In particular, for $l_1 = -l_2 = l$ we superimpose two optical vortices with opposite topological charges and equal intensity, and the

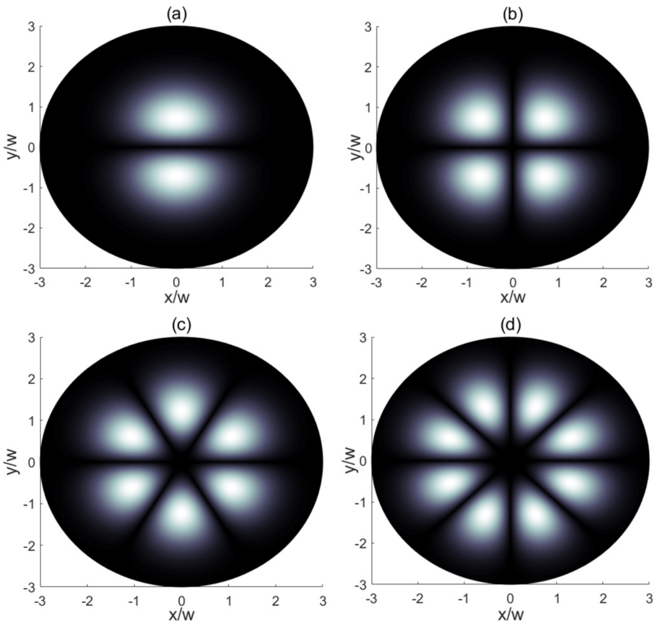

FIG. 8. Intensity distributions in arbitrary units for the beam $\Omega_1(z)$ defined by Eq. (55) generated by combining two vortex beams with vorticities (a) $l_1 = 1$, $l_2 = -1$, (b) $l_1 = 2$, $l_2 = -2$, (c) $l_1 = 3$, $l_2 = -3$, and (d) $l_1 = 4$, $l_2 = -4$. The selected parameters are the same as Fig. 6. The intensity distribution of the field $\Omega_2(z)$ is identical to the intensity distribution of the field $\Omega_1(z)$ shown in this figure.

azimuthal dependence is given by $e^{il\phi} + e^{-il\phi} = 2\cos(l\phi)$. This corresponds to the flowerlike "petals" intensity structures demonstrated in Fig. 8. Note that such a flowerlike structure is not called a vortex, although it has a zero intensity at the center [68,69].

Let us now assume $\alpha_1 = \alpha_2 = \alpha$, $\gamma_{eg_1} = \gamma_{eg_2} = \gamma$, and $\delta_1 = \delta_2 = 0$. At propagation distances exceeding the absorption length $z \gg L_{abs}$ all exponential terms vanish in Eqs. (55) and (56) and we get

$$\Omega_1(z \gg L_{abs}) = e^{-r^2/w^2}\left[|c_2|^2\varepsilon_1\left(\frac{r}{w}\right)^{|l_1|}e^{il_1\phi} - c_1c_2^*\varepsilon_2\left(\frac{r}{w}\right)^{|l_2|}e^{il_2\phi}\right], \tag{57}$$

$$\Omega_2(z \gg L_{abs}) = e^{-r^2/w^2}\left[-c_1^*c_2\varepsilon_1\left(\frac{r}{w}\right)^{|l_1|}e^{il_1\phi} + |c_1|^2\varepsilon_2\left(\frac{r}{w}\right)^{|l_2|}e^{il_2\phi}\right]. \tag{58}$$

Therefore, if the two incident vortex fields are nonzero, for $z \gg L_{abs}$ both vortex beams experience no absorption as the atoms comprising the medium are converted to their dark states defined by Eq. (19). It is noteworthy that there is an even more favorable scenario for the lossless propagation of both vortex beams. Assuming that $\Omega_1(0) = \Omega_2(0) = \Omega = \varepsilon(\frac{r}{w})^{|l|}e^{-r^2/w^2}e^{il\phi}$ and choosing the values of c_1 and c_2 such that $c_1 = -c_2 = \frac{1}{\sqrt{2}}$, one arrives at $\Omega_1(z) = \Omega_2(z) = \Omega(z)$. Under this condition the atoms are in their dark states from the very beginning, the medium becomes completely transparent to both vortex beams, and the fields propagate without losses as in free space. Such an analysis for generation and propagation of composite optical vortices can be extended to the $(n+1)$-level schemes when all n laser fields are present at the entrance to the atomic medium.

VI. CONCLUDING REMARKS

We have analyzed the propagation dynamics of two (three) component laser pulses with OAM interacting with atoms in the Λ (tripod) atom-light coupling schemes. The quantum system is initially prepared in a coherent superposition of two (three) lower levels. If a vortex beam acts on one transition of the Λ (tripod) system, an extra light beam can be nonlinearly generated with the same OAM number as the initially injected structured light. We have also extended the analysis to a $(n+1)$-level phaseonium medium for the n-component generation of the twisted light beams. The lossless propagation of generated vortex light beams has been also considered. It has been shown that at the propagation distances exceeding the absorption length the system goes to a linear superposition of $n-1$ dark states leading to the transparency of the medium to the n-component optical vortices.

It has been recently shown that a double-Λ scheme can be employed for the exchange of optical vortices based on EIT [55]. In the double-Λ scheme there should be two additional control lasers of larger intensity to assure the exchange of optical vortices. On the other hand, in the current proposal one does not need the strong atom-light interaction as we are dealing with small intensities ($|\Omega_i| \ll \gamma_{eg_i}$). It is only needed that a medium is initially coherently prepared in a superposition of atomic lower levels. The losses in both schemes are similar and take place mostly at beginning of the medium within the absorption length. The losses disappear when the light pulses propagate deeper through the medium.

We have also considered a situation where both vortex beams Ω_1 and Ω_2 are present at the beginning of the medium of the Λ-type atoms. When the two vortex beams are incident on the medium, they can create two composite beams with new vortices. Different cases for the appearance of composite vortices have been explored, and the situations for absorptionless propagation of composite vortices are discussed. We have also extended the model for generation of composite optical vortices to the $(n+1)$-level structures.

The coherent superposition of the ground states employed in this paper can be realized experimentally using the fractional or partial STIRAP in which only a controlled fraction of the population is transferred to the target state [64]. The creation of a quantum superposition of metastable states out of a single initial state in a robust and controlled way has been shown to be possible in a four-state system by using a sequence of three pulses [30,31]. Such a technique is based on the existence of two degenerate dark states and their interaction. The mixing of the dark states can be controlled by changing the relative delay of the pulses, and thus an arbitrary superposition state can be generated. Such a method for creation of coherent superpositions can be generalized to N level schemes.

The Λ (tripod) level scheme containing two (three) ground states and one excited state may be implemented experimentally, for example, using the ^{87}Rb atoms. The excited level $|e\rangle$ can then correspond to the $|5P_{1/2}, F = 1, m_F = 0\rangle$ state. The lower states $|g_1\rangle$ and $|g_2\rangle$ (and $|g_3\rangle$) can be attributed to the $|5S_{1/2}, F = 1, m_F = 1\rangle$ and $|5S_{1/2}, F = 1, m_F = -1\rangle$ (and $|5S_{1/2}, F = 2, m_F = 0\rangle$) [70].

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[1] S. E. Harris, Phys. Today **50**(7), 36 (1997).
[2] I. E. Mazets, Phys. Rev. A **71**, 023806 (2005).
[3] F. T. Hioe, Phys. Rev. A **78**, 063807 (2008).
[4] J. R. Csesznegi and R. Grobe, Phys. Rev. Lett. **79**, 3162 (1997).
[5] M. D. Lukin, Rev. Mod. Phys. **75**, 457 (2003).
[6] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005).
[7] E. Paspalakis and P. L. Knight, Phys. Rev. A **66**, 015802 (2002).
[8] G. Juzeliūnas and P. Öhberg, Phys. Rev. Lett. **93**, 033602 (2004).
[9] S. E. Harris, J. E. Field, and A. Imamoğlu, Phys. Rev. Lett. **64**, 1107 (1990).
[10] L. Deng, M. G. Payne, and W. R. Garrett, Phys. Rev. A **58**, 707 (1998).
[11] H. Wang, D. Goorskey, and M. Xiao, Phys. Rev. Lett. **87**, 073601 (2001).
[12] H. Kang and Y. Zhu, Phys. Rev. Lett. **91**, 093601 (2003).
[13] S. E. Harris, Phys. Rev. Lett. **72**, 52 (1994).
[14] E. Cerboneschi and E. Arimondo, Phys. Rev. A **52**, R1823 (1995).
[15] R. G. Unanyan, J. Otterbach, M. Fleischhauer, J. Ruseckas, V. Kudriašov, and G. Juzeliūnas, Phys. Rev. Lett. **105**, 173603 (2010).
[16] J. Ruseckas, V. Kudriašov, G. Juzeliūnas, R. G. Unanyan, J. Otterbach, and M. Fleischhauer, Phys. Rev. A **83**, 063811 (2011).
[17] Q.-Q. Bao, X.-H. Zhang, J.-Y. Gao, Y. Zhang, C.-L. Cui, and J.-H. Wu, Phys. Rev. A **84**, 063812 (2011).
[18] J. Ruseckas, V. Kudriašov, I. A. Yu, and G. Juzeliūnas, Phys. Rev. A **87**, 053840 (2013).
[19] M.-J. Lee, J. Ruseckas, C.-Y. Lee, V. Kudriašov, K.-F. Chang, H.-W. Cho, G. Juzeliūnas, and I. A. Yu, Nat. Commun. **5**, 5542 (2014).
[20] R. Grobe, F. T. Hioe, and J. H. Eberly, Phys. Rev. Lett. **73**, 3183 (1994).
[21] M. Fleischhauer and A. S. Manka, Phys. Rev. A **54**, 794 (1996).
[22] Y. Wu and L. Deng, Phys. Rev. Lett. **93**, 143904 (2004).
[23] Y. Wu and L. Deng, Opt. Lett. **29**, 2064 (2004).
[24] D. F. Phillips, A. Fleischhauer, A. Mair, R. L. Walsworth, and M. D. Lukin, Phys. Rev. Lett. **86**, 783 (2001).
[25] C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, Nature (London) **409** (2001).
[26] M. D. Lukin and A. Imamoglu, Nature (London) **413** (2001).
[27] G. Juzeliūnas and H. J. Carmichael, Phys. Rev. A **65**, 021601(R) (2002).
[28] Y.-F. Hsiao, P.-J. Tsai, H.-S. Chen, S.-X. Lin, C.-C. Hung, C.-H. Lee, Y.-H. Chen, Y.-F. Chen, I. A. Yu, and Y.-C. Chen, Phys. Rev. Lett. **120**, 183602 (2018).
[29] L. Ma, O. Slattery, and X. Tang, J. Opt. **19**, 043001 (2017).
[30] R. G. Unanyan, M. Fleischhauer, B. W. Shore, and K. Bergmann, Opt. Commun. **155**, 144 (1998).
[31] R. G. Unanyan, B. W. Shore, and K. Bergmann, Phys. Rev. A **59**, 2910 (1999).
[32] E. Paspalakis and P. L. Knight, J. Opt. B: Quantum Semiclass. Opt. **4**, S372 (2002).
[33] J. Ruseckas, A. Mekys, and G. Juzeliūnas, J. Opt. **13**, 064013 (2011).
[34] E. A. Korsunsky and D. V. Kosachiov, Phys. Rev. A **60**, 4996 (1999).
[35] X.-J. Liu, H. Jing, X.-T. Zhou, and M.-L. Ge, Phys. Rev. A **70**, 015603 (2004).
[36] M. G. Payne and L. Deng, Phys. Rev. A **65**, 063806 (2002).
[37] J. Ruseckas, V. Kudriašov, A. Mekys, T. Andrijauskas, I. A. Yu, and G. Juzeliūnas, Phys. Rev. A **98**, 013846 (2018).
[38] C. Ding, J. Li, Z. Zhan, and X. Yang, Phys. Rev. A **83**, 063834 (2011).
[39] H. R. Hamedi, J. Ruseckas, and G. Juzeliūnas, J. Phys. B: At., Mol., Opt. Phys. **50**, 185401 (2017).
[40] E. Paspalakis, N. J. Kylstra, and P. L. Knight, Phys. Rev. A **65**, 053808 (2002).
[41] E. Paspalakis and Z. Kis, Phys. Rev. A **66**, 025802 (2002).
[42] L. Allen, M. J. Padgett, and M. Babiker, Prog. Opt. **39**, 291 (1999).
[43] M. Padgett, J. Courtial, and L. Allen, Phys. Today **57**(5), 35 (2004).
[44] M. Babiker, W. L. Power, and L. Allen, Phys. Rev. Lett. **73**, 1239 (1994).
[45] V. E. Lembessis and M. Babiker, Phys. Rev. A **82**, 051402 (2010).
[46] V. E. Lembessis, D. Ellinas, M. Babiker, and O. Al-Dossary, Phys. Rev. A **89**, 053616 (2014).
[47] Q.-F. Chen, B.-S. Shi, Y.-S. Zhang, and G.-C. Guo, Phys. Rev. A **78**, 053810 (2008).
[48] D.-S. Ding, Z.-Y. Zhou, B.-S. Shi, X.-B. Zou, and G.-C. Guo, Opt. Lett. **37**, 3270 (2012).
[49] G. Walker, A. S. Arnold, and S. Franke-Arnold, Phys. Rev. Lett. **108**, 243601 (2012).
[50] N. Radwell, T. W. Clark, B. Piccirillo, S. M. Barnett, and S. Franke-Arnold, Phys. Rev. Lett. **114**, 123603 (2015).
[51] S. Sharma and T. N. Dey, Phys. Rev. A **96**, 033811 (2017).
[52] H. R. Hamedi, V. Kudriašov, J. Ruseckas, and G. Juzeliūnas, Opt. Express **26**, 28249 (2018).
[53] Z. Dutton and J. Ruostekoski, Phys. Rev. Lett. **93**, 193602 (2004).
[54] J. Ruseckas, G. Juzeliūnas, P. Öhberg, and S. M. Barnett, Phys. Rev. A **76**, 053822 (2007).
[55] H. R. Hamedi, J. Ruseckas, and G. Juzeliūnas, Phys. Rev. A **98**, 013840 (2018).
[56] T. Wang, L. Zhao, L. Jiang, and S. F. Yelin, Phys. Rev. A **77**, 043815 (2008).
[57] J. Ruseckas, A. Mekys, and G. Juzeliūnas, Phys. Rev. A **83**, 023812 (2011).

[58] R. Pugatch, M. Shuker, O. Firstenberg, A. Ron, and N. Davidson, Phys. Rev. Lett. **98**, 203601 (2007).
[59] D. Moretti, D. Felinto, and J. W. R. Tabosa, Phys. Rev. A **79**, 023825 (2009).
[60] M. O. Scully, Phys. Rep. **219**, 191 (1992).
[61] M. O. Scully, Phys. Rev. Lett. **55**, 2802 (1985).
[62] M. Fleischhauer, C. H. Keitel, M. O. Scully, C. Su, B. T. Ulrich, and S.-Y. Zhu, Phys. Rev. A **46**, 1468 (1992).
[63] J. H. Eberly, A. Rahman, and R. Grobe, Phys. Rev. Lett. **76**, 3687 (1996).
[64] N. V. Vitanov, A. A. Rangelov, B. W. Shore, and K. Bergmann, Rev. Mod. Phys. **89**, 015006 (2017).
[65] N. A. Chaitanya, S. C. Kumar, K. Devi, G. K. Samanta, and M. Ebrahim-Zadeh, Opt. Lett. **41**, 2715 (2016).
[66] S. Rebić, D. Vitali, C. Ottaviani, P. Tombesi, M. Artoni, F. Cataliotti, and R. Corbalán, Phys. Rev. A **70**, 032317 (2004).
[67] J. Ruseckas, G. Juzeliūnas, P. Öhberg, and M. Fleischhauer, Phys. Rev. Lett. **95**, 010404 (2005).
[68] S. Franke-Arnold, J. Leach, M. J. Padgett, V. E. Lembessis, D. Ellinas, A. J. Wright, J. M. Girkin, P. Ohberg, and A. S. Arnold, Opt. Express **15**, 8619 (2007).
[69] S. M. Baumann, D. M. Kalb, L. H. MacMillan, and E. J. Galvez, Opt. Express **17**, 9818 (2009).
[70] L.-G. Si, W.-X. Yang, and X. Yang, J. Opt. Soc. Am. B **26**, 478 (2009).